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Israel Lyons Jun$^{r}$

Nov. 1762.

*Sir George Shuckburgh Bar$^{t}$*

**N$^{o}$ 4**

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# Hiſtoria Fluxionum,

Sive Tractatus Originem &
PROGRESSUM
Peregregiæ Iſtius Methodi
BREVISSIMO
COMPENDIO
(Et quaſi Synopticè) Exhibens.

K

Per *JOSEPHUM RAPHSONUM* (τοῦ νῦν μακαρείτου)
A. M. & *R. S. S.*

*LONDINI:*
Typis *Pearſonianis*, Proſtant Venales apud *Richardum Mount*,
in loco vocato *Tower-Hill.* MDCCXV.

## PRÆMONITIO LECTORI.

*Quædam Errata Præli prætermiſſa, propter morbum & mortem Authoris, antequam Opus totaliter peractum fuit; quæ emendare Lector deſideratur.*

BRITISH MUSEUM

Illuſtri *Societati Regali Londinenſi*,

AD

Philoſophiam Promovendam,

AB AUGUSTISSIMO

Principe *CAROLO* II.

FUNDATÆ, ET

JAM *SUB* AUSPICIIS

Potentiſſimi Regis,

GEORGII

FLORENTI.

Opuſculum Hocce, Complectens.

Qua par eſt Veneratione,

D. D. C. Q.

Bibliopola.

# Præfatio Autoris.

BRITISH MUSEUM

*INſequentem Hiſtoriam Conſcripturus, duo potiſſimùm agere mecum Conſtituiſſem.*

1°. *Inventiones præcipuas in hoc genere Primis ſuis Authoribus Speciatim aſſerere & Vindicare; & Imprimis D.* Neutono *Inventori Longè, & tempore primo, & Præſtantia Inventionum, præcellentiſſimo.*

2°. *Ut aditus ad hæcce facilius & Explicatius Tyronibus pateſieret, adeòq; ut cuivis Introductionis Loco ad* Neutoniana, Cheynæana, *aliorumque inventa & congeſta in hoc genere inſerviret: Si hæc quadantenus attigerim, Quod mihi propoſui conſequutus ſum.*

*Dum hæc ſub Prælo eſſent, prodiit Liber, ſub titulo* Commercii Epiſtolici, *R. S. juſſu Impreſſus, ea quæ ſparſim in* Walliſio *ſunt, multaſq; alias nondum Evulgatas Epiſtolas inter* Collinſium Noſtratem, Neutonum, Leibnitium *alioſq; Complectens. In quibus Nuda Rei veritas, quoad diſceptationem inter* Leibnitium & Neutonum, *de Prima Fluxiorum Inventione, Luce Meridiana clarius apparet. Caput ultimum, ſeu Concluſio, Excerpta quædam ex illo Continet; Cum ejuſdem editio perpuſilla ſit, & nulla Exemplaria in Bibliopolarum manus pervenerint.*

*Hoc, quidquid eſt opellæ, ut Lectorem diutius morari deſinam, cenſuræ ſuæ permittimus.*

J. R.

# CAP. I.

## *De Inventore, vel Inventoribus Fluxionum.*

POST varias *Exhauſtionum* & *Indiviſibilium* methodos ab *Archimede* quidem Inceptas, & dudum poſteà a *Cavallerio* reſuſcitatas & promotas, poſtque Arithmeticam infinitorum a *Walliſio* feliciter quidem ſatis excogitatam, ante verò varia illa infinitarum Serierum Specimina a *Nicholao Mercatore*, & *Jacobo Gregorio* per Diviſionem in Speciebus exhibita, poſtq; *Theoremata* illa præclara *Tetragoniſtica Barovii* in lectionibus ſuis ſparſim prodita, quorumq; partem aliquam (uti jure optimo queritur D. *Craigius*) pro ſuis edidit, & actis erud. inſeruit D. *Tſchirnhauſius*, patet celeberrimum virum D. *Iſaacum Neutonum* circa Annum (ut *Walliſius* ait, & ex MSS. antiquis colligitur) 1665, vel 1666, in Methodum illam peregregiam Fluxionum incidiſſe, cujuſq; ſpecimina quædam dedit in *Analyſi* ſua *per æquationes numero terminorum infinitas* quam *Barrovius* Anno 1699, ad D. *Johannes Collinſium* miſit, ut & in Epiſtola ſua 10 *Decem.* 1672, ad *Collinſium* miſſa. Circa initium quidem Anni 1670, *Collinſius* literis ad Clariſſimum Virum D. *Jacobum Gregorium* conſcriptis ſignificavit D. *Iſaacum Newtonum* Methodi Quadraturarum generalis compotem eſſe, uti teſtatus eſt D. *Dav. Gregorius* in Exercitatione ſua Geometrica Anno 1684, publicata, pag. tertia, & inter cæteros quidem, Anno 1676, eandem in Epiſtola quadam ad Celeberrimum Virum *G. G. Leibnitium* miſit, & ejus beneficio Analyſin ad omnia fere Problemata ſeſe extendere ſignificavit, ſed abſq; aliis quibuſdam methodis non omnino univerſalem evadere.

*(a)* Eodem porro Anno, ſcil 1676. D. *Newtonus* in Epiſtola Die 24. *Octobris* data, ad Dom. *Oldenburgium* tum temporis Reg. Soc. Secretarium ſcripſit *(b)* ſe tunc *compotem fuiſſe methodi cujuſdam qua* Sluſius *curvarum tangentes ducit, ſed quæ* Sluſianâ *generalior eſſet, in hac enim non hæreri ad æquationes radicalibus unam vel utramq; indefinitam quantitatem (ordinatam ſcil. & abſciſſam) involventibus utcunque adſectas; ſed abſq; aliquâ talium æquationum reductione (quæ opus plerumq; redderet immenſum) tangentem confeſtim duci: Nec in tangentibus tantum & maximis minimiſq; determinandis, ſed etiam in Quadratura Curvarum, & queſtionibus quibuſdam aliis hanc methodum locum habere. Nam & ejus beneficio ſe generalia quæ-*

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(a) *Vid.* Wall. *Oper.* Vol. II. p. 390.

(b) *Hicce tractatus flammis periit, uti* p. 390. Vol. II. Wall. *videre eſt Vid.* p. 391.

*dum de Quadratura Curvarum Theoremata inveniſſe.* *(c)* Et ibidem Theorema ſubnectit pro Binomiis univerſalibus, quod *Primum* vocat, (hoc eſt) primum Seriei Theorematum ad infinitum, ſeu *infinitinomium*, in ſuo uniuſcujuſq; genere, pergentis; & rurſus alias horum Theorematum pro Infinitinomiis ſeries ſeu progreſſiones ad infinitum uſq; conſcendentium patefacit, uti ſuo loco fuſius patebit.

Problema in quo fundabatur hæcce methodus literis quidem tranſpoſitis ad hunc modum celabat *(6accdæ*$^{13}$*eff*$^{7}$*i*$^{3}$*l*$^{9}$*n*$^{4}$*o*$^{4}$*qrr*$^{4}$*s*$^{9}$*t*$^{12}$*vx)* quæq; ordinatæ hanc conficiunt ſententiam, *Data æquatione Fluentes quotcunque quantitates involvente, invenire Fluxiones; & vice verſa.* Hoc artificio deducebat Problemata ad æquationes fluxionales, & viciſſim ex hujuſmodi æquationibus (vel regrediendo, vel per Quadraturam Curvilinearum, methodumve ſerierum, aut alias artes) fluentes deducebat.

Methodos enim ad æquationes Fluxionales tunc temporis, Ann. ſcil. 1676, accomodaverat, hoc eſt ad *eas æquationes* (ut verbis *Cheynæanis* utar) *ad Fluentes revocandas in quibus Fluentes & earum Fluxiones ita confunduntur ut facile (ſervatis iiſdem indeterminatis) ſeparari nequeant.*

Methodos deniq; univerſaliores ad *Comparationes Curvarum* quæ Geometrice quadrari non poſſunt, eodem tempore concinnaſſet & *(d)* cum amicis quibuſdam communicârat uti e prædicto Cl. *Walliſii* operum volumine ſecundo (ut & abunde e tractatu de Quadratura Curvarum) patet; adeo ut vix dici poſſit, anne aliorum omnium in hiſce inventiones anticipavit, an vero fere omnibus (per communicationem ſuorum diu antequam edita fuiſſent) anſam ſimilia excogitandi, vel potius eadem *Neutoniana* alia forma edendi, ſubminiſtraverit.

Mente *Newtoni* ex Epiſtolis ejus percepta, Vir Celeberrimus D. *Leibnitius* Anno proximo reſcripſit, ſe in parilem Methodum incidiſſe (ut ex ejus Epiſtola in tertio Volumine Operum *Walliſii* impreſſa videre licet,) cujus tamen explicationem (in cæteris ſuis inventis publicandis non raro nimis præproperus, uti in Actis Erud. plus ſemel conſpici queat,) ad Annum uſq; 1684, diſtulerit.

Tandem vero *Newtonus* Anno 1704, Tractatulum quem circa Annum 1676, ex Tractatu antiquiore deſcripſerat, redivivum publici juris fecerit; quemq; circiter Annem 1691. Vir Clariſſ. *Halleius* & ego *Cantabrigiæ* in manibus habuiſſemus, tum quidem prælo paratum, & perlegendo (ab iis quibus eundem mutuo dederat) obtritum, quemq; poſtea reviſendum repoſtulaverit, & ad annum uſq; prædictum ejuſdem publicationem diſtulerit.

*Newtonus* ergò Ann. 1665, & 1666, methodum hance adinvenerit, & Anno 1676, notitias hujuſce ſuæ Inventionis inter cæteros *(e)* cum *Leibnitio* communicaverit, (uti modo diximus) qui multum poſtea ſcil. Menſe *Octobris* 1684, calculi ſui *Differentialis* principia in act. Erudit.

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(c) *Vid.* Paragraph p. 393. *incip. ſub finem* Epiſt. An. 1676.

(d) *Pag.* 323. *ut ſupra.*

(e) *Princ. Nat. Ph. Math.* p. 253.

publicavit, hoc eſt, non niſi 19 Annos poſt eundem a *Neutono* inventum, & 8 Annos poſtquam notitiam hancce ſibi habuit communicatam.

*Barrovium* porro huic calculo accendiſſe aliquâ ex parte facem quibuſdam viſum eſt ; & hoc etiam fateri videtur *J. Bernoulli* in act. Erud. Menſe *Januarij* Ann. 1691. hiſce verbis, *Quanquam ut verum fatear, qui calculum* Barrovianum (*quem decennio ante in Lectionibus ſuis Geometricis adumbravit Author cujuſq; Specimina ſunt tota illa Propoſitionum inibi contentarum farrago*) *intellexerit, alterum a Domino* Leibnitio *inventum ignorare vix poterit; utpote qui in priore illo fundatus eſt, & niſi forte in differentialium notatione, & operationis aliquo compendio ab eo non differt.* Poſtea vero (Menſe ſcil. *Junij* ejuſdem Anni) ne nimium hic conceſſum videretur, audi eundem loquentem de Inventione hacce (uti ille vult) *Leibnitianâ* ; *cæterum in hiſce Problematîs omnibus quæ quis nequicquam alia tentet methodo, calculi* Leibnitiani *eximium & ſingularem uſum eſſe comperi, ut ipſum propterea inter primaria ſœculi noſtri inventa cenſendum eſſe æſtimem. Quanquam enim ut nuper innui, anſam huic dediſſe credam calculum* Barrovij *qualem appello, qui ab hujus viri tempore paſſim fere apud Geometras præſtantiores invaluit, quemq; etiamnum nobiliſſimo* Tſchirnhauſio *ſolennem eſſe video : Hoc tamen non eo intelligendum eſt, quaſi utiliſſimi inventi dignitatem ullatenus elevare, aut celeberrimi viri laudi quicquam detrahere & aliis adſcribere cupiam ; & ſi quæ conferenti mihi utrumq; inter illos viſa eſt affinitas, ea major non eſt, quam quæ faciat ut uno intellecto, ratio alterius facilius comprehendatur, dum unus ſuperfluas & mox delendas quantitates adhibet, quas alter compendio omittit ; de cætero namque compendium iſthoc tale eſt, quod naturam rei prorſus mutat, facitq; ut infinita per hunc præſtari poſſunt, quæ per alterum nequeunt : Præterquam quod ipſum hoc compendium reperiſſe utiq; non erat cujuſvis, ſed ſublimis ingenii & quod Authorem maxime commendat.*

Ut res ipſa autem quam clariſſime appareat, mecum conſtitui ipſa primorum Authorum inventa præcipua ſuis quidem verbis ſynoptice exhibere, & additis hinc inde quibuſdam explicatiunculis, quo melius etiam praxi tractatulus hicce accomodetur, rem totam ſub nomine *Hiſtoriæ* concinnatam, addita inſuper, (in quantum poſſum) inventionum etiam *Chronologia*, publici juris facere.

# CAP. II.

## *De* NOTATIONE, &c.

QUali ab initio hujuſce ſui inventi notatione uſus fuerit Cl. *Neutonus* non adeo facile conſtat; quamvis nonnulla ſint quæ me movent ut ſuſpicer illum in eandem hancce punctationis Methodum qua poſteà uſus eſt, etiam ab initio incidiſſe; licet in Lemmate ſuo pag. 256. *Princip. Phil. Nat. Math.* in quo hujuſce methodi algorithmum tradit, alia in demonſtratione ibidem allata utitur, deſignando Fluentes per literas Alphabeti majuſculas *A*, *B*, *C*, *&c.* & earundem Fluxiones per minuſculas *a*, *b*, *c*, *&c.* reſpective. *Barrovii* porro notationem in ſua Methodo quadraturarum etiam in differentiis indeterminatarum denotandis inſecutus eſt D. *Nieuwentijt*, in Analyſi ſua infinitorum, aſſignando ſcil. vocales *a*, *e*, *i*, *o*, *u*, pro differentiis (ſeu Fluxionibus) ipſarum Indeterminatarum *x*, *y*, *&c.* quaſq; partes *infiniteſimas* vocat.

*Leibnitius* diverſam ab hiſce ſibi elegerit, quamq; ſequuti ſunt Fratres ambo *Bernoullij*, & *Germani*, a quibus etiam eandem acceperunt & *Galli* quotquot huic rei navarunt operam; Indeterminatæ cujuſvis per literas $x$, $y$, $z$, expreſſas, differentiam (ſeu *Fluxionum*) primam ponit, $dx$, $dy$, $dz$; ſecundam vero $ddx$, $ddy$, $ddz$, tertiam $dddx$, $dddy$, $dddz$, ut & $\overline{dx}|^3$, $\overline{dy}|^3$, $\overline{dz}|^3$, & ſic porro; hancce etiam notationem inter Britannos ſequutus eſt Geometra clariſſimus *Johann. Craigius.* Sed (quæ jam apud *Anglos* plerumq; obtinet) differentias haſce, quas aptiſſimo vocabulo, & ex natura generationis earundem per motum continuum petito, *Fluxiones* nominavit, & deſignavit per punctationes, *Celeberrimus*, & *Primarius* Doctrinæ hujuſce Author, D. *Neutonus*, viz. $\dot{x}$, $\dot{y}$, $\dot{z}$, & earundem *Fluxiones* ſecundas per punctationem duplicem ſic $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, tertias, ſic $\dddot{x}$, $\dddot{y}$, $\dddot{z}$, & ſic porro ad infinitum; hiſce præterea adjicit alias ſeries quantitatum infinite magnarum ſcil. $\overset{\prime}{x}$, $\overset{\prime}{y}$, $\overset{\prime}{z}$, quarum ut *Fluxiones Primæ* conſiderari poſſint quantitates ipſæ finitæ $x$, $y$, $z$, ſed & ipſæ porro infinitæ uti *Fluxiones* aliarum altioris infinitatis gradus æſtimari queant, ſcil. (a) harum $\overset{\prime\prime}{x}$, $\overset{\prime\prime}{y}$, $\overset{\prime\prime}{z}$, *&c.* & hæ rurſus aliarum $\overset{\prime\prime\prime}{x}$, $\overset{\prime\prime\prime}{y}$, $\overset{\prime\prime\prime}{z}$, *&c.* & ſic in infinitum, adeo ut Seriem hujuſmodi conſtituunt, *viz.* $\overset{''''}{z}, \overset{'''}{z}, \overset{''}{z}, \overset{'}{z}, z, \dot{z}, \ddot{z}, \dddot{z}, \ddddot{z}$, &c. utrinque in Infinitum continuandam, *quarum quælibet poſterior eſt Fluxio præcedentis, & quælibet prior eſt Fluens quantitas fluxionem habens ſubſequentem.* Eodem modo hanc Seriem pariter conſimilem ibidem memorat ſcil. $\sqrt{\overset{''}{az-zz}}, \sqrt{\overset{'}{az-zz}}, \sqrt{az-zz}, \sqrt{\overset{\cdot}{az-zz}}, \sqrt{\overset{\cdot\cdot}{az-zz}}, \sqrt{\overset{\therefore}{az-zz}}$, &c. ut et $\overset{''}{\frac{az+zz}{a-z}}, \overset{'}{\frac{az+zz}{a-z}}, \frac{az+zz}{a-z}, \overset{\cdot}{\frac{az+zz}{a-z}}, \overset{\cdot\cdot}{\frac{az+zz}{a-z}}$, &c. ad Inf. ubi ( inquit ) *quantitas quælibet prior eſt ut Area Figuræ Curvilineæ cujus Ordinatim Applicata rectangula eſt quantitas poſterior, & abſciſſa eſt* z *: nti* $\sqrt{\overset{'}{az-zz}}$ *area erit Curvæ cujus Ordinatæ eſt* $\sqrt{az-zz}$ *& abſciſſa* z. Sicque punctum (in ſenſu ſaltem Laxiori acceptum) poſſit conſiderari uti Fluxio Lineæ, Linea uti Fluxio Plani, Planum uti Fluxio Solidi, Solidum Finitum uti Fluxio Infiniti, & Solidum etiam ex una parte Infinitum uti Fluxio Solidi Infinitioris, ſeu infinite majoris, & ſic ad Infinitum; quod, in diſſertatiunculis quibuſdam, ad Calcem hujuſce Tractatus, accuratius illuſtratum dabimus. Sicque etiam hujuſmodi Series viz.

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(a) Pag. 171. *Optic. Edit. Angl.*

$$\frac{\dot{x}}{x}, \frac{\dot{x}}{x'}, \frac{\dot{x}}{x''}, \frac{\dot{x}}{x'''}, \text{\&c. } \frac{\ddot{x}}{x'}, \frac{\ddot{x}}{x''}, \frac{\ddot{x}}{x'''}, \text{\&c. } \frac{\dddot{x}}{x'}, \frac{\dddot{x}}{x''}, \frac{\dddot{x}}{x'''}, \text{\&c. ad Inf.}$$

ut et aliæ Series Infinitæ, quantitates ſui generis repreſentare poſſint. Sin autem hoſce & conſimiles Fluxionum gradus Lector quivis ægre admiſerit, ſciat Auctores Celeberrimos *Neutonum* & *Leibnitium*, eoſdem jamdiu (ſenſu licet valde diverſo) uti poſtea videre eſt, admiſiſſe, ipſe etenim *Leibnitius* in Act. Erud. pro Menſe *Febr.* 1689, in *Tentamine de Motuum Cæleſtium Cauſis*, hæc habet, viz. *Aſſumſi inter demonſtrandum quantitates incomparabiliter parvas, verbi gratia, differentiam duarum quantitatum communium ipſis quantitatibus incomparabilem. Sic enim talia, ni fallor, Lucidiſſime exponi poſſunt, itaque ſi quis nolit adhibere* Infinite parvas, *poteſt aſſumere tam parvas quam ſufficere judicat, ut ſint incomparabiles, & errorem nullius momenti, imo dato minorem, producant. Quemadmodum Terra pro puncto, ſeu diameter Terræ pro linea infinite parva habetur, reſpectu Cœli; Sic demonſtrari poteſt, ſi anguli latera habeant baſin ipſis incomparabiliter minorem, angulum comprehenſum fore recto incomparabiliter minorem, & differentiam laterum fore ipſis differentibus incomparabilem; item differentiam ſinus totius, ſinus complementi, & ſecantis, fore differentibus incomparabilem, ita differentiam ſinus, Chordæ, Arcus, & Tangentis. Unde cum hæ ſint ipſæ infinite parvæ, erunt differentiæ* infinities infinite parvæ, *& ſinus verſus etiam erit* infinities infinite parvus, *adeoque recto incomparabilis. Et Infiniti ſunt gradus tam Infinitorum, quam infinite parvorum. Et poſſunt adhiberi triangula communia inaſſignabilibus illis ſimilia, quæ in Tangentibus, maximiſque & minimis, & explicanda curvedine linearum uſum habent* *maximum*; *item in omni pene* tranſlatione Geometriæ ad Naturam; *nam ſi motus exponatur per lineam communem quam dato tempore mobile abſolvit, impetus ſeu velocitas exponetur per lineam infinite parvam, & ipſum Elementum velocitatis, quale eſt gravitatis ſolicitatio, vel conatus centrifugus, per lineam infinities infinite parvam, &c.* Et eundem fere ad modum magis etiam naturaliter *Neutonus* diu antea pag. 165 Optic. prædict. *Quantitates Mathematicas non ut ex partibus quam minimis conſtantes, ſed ut motu continuo deſcriptas hic conſidero, lineæ deſcribuntur ac deſcribendo generantur non per appoſitionem partium, ſed per motum continuum punctorum, ſuperficies per motum linearum, ſolida per motum ſuperficierum, anguli per rotationem laterum, tempora per fluxum continuum, & ſic in cæteris. Hæ geneſes in rerum natura locum vere habent, & in motu corporum quotidie cernuntur.* Sed ut ad notationes redeam; Vir celeberrimus *Newtonus* invexit quaſdam alias (nam dicit *ſe neceſſum habuiſſe mutare*) operationibus hujuſce calculi commodius perficiendis, quæ jam diu apud Mathematicos invaluerunt; quas videre licet apud *Walliſium*, pag. 315. vol. 9. Edit. noviſſimæ, uti ſubſtituendo.

*pro* $\frac{1}{x} \cdots\cdots\cdots x^{-1}$

$\frac{1}{xx} \cdots\cdots\cdots x^{-2}$

$\frac{1}{x^3} \cdots\cdots\cdots x^{-3}$ &c.

$\sqrt{x} \cdots\cdots x^{\frac{1}{2}}$

$\sqrt{x^3} \cdots\cdots x^{\frac{3}{2}}$

$\sqrt[3]{x} \cdots\cdots x^{\frac{1}{3}}$

$\frac{1}{\sqrt{x}} \cdots\cdots x^{-\frac{1}{2}}$

$\frac{1}{x^{\nu}} \cdots\cdots\cdots x^{-\nu}$

$\frac{a}{x} \cdots\cdots\cdots ax^{-1}$

*& pro Quadrato* $\overline{e+fx},| \cdots \overline{e+fx}|^{2}$

$\sqrt{\overline{aa-xx}} \cdots \overline{aa-xx}|^{\frac{1}{2}}$

$\frac{1}{\sqrt{\overline{aa-xx}}} \cdots \overline{aa-xx}|^{-\frac{1}{2}}$

$\frac{dx}{\overline{e+fx}|^{2}} \cdots dx \times \overline{e+fx}|^{-2}$ &c.

Huc etiam referri poſſunt operationes iſtæ *Newtonianæ* per Indices peragendæ, *ſcil.*

1. Quod Additio Indicum æquivalet Multiplicationi Poteſtatum uti $x^2 \times x^3 = x^{2+3} = x^5$.

2. Subtractio Indicum Diviſioni Dignitatum æquipollet; ſic $\frac{x^3}{x^2} = x^{3-2} = x^1 = x$.

3. Multiplicatio Indicum æquivalet Involutioni Dignitatum quibus præfixi ſunt, ſic $x^{3\times 3}$ *Cub.* $x^3 = x^9$.

4. Diviſio Indicum æquipollet Evolutioni, *e. g.* $x^{\frac{6}{3}} = \sqrt[3]{x^6}$ quia $x^{2\times 3} =$ *Cub.* $x^2 = x^6$.

Sed & porro aliam Notationis methodum invexit in Tractatu ſupra laudato de Quadraturis, maximam & commodiſſimam Elucidationem in hiſce præ ſe ferentem, viz. ponendo literam aliquam majuſculam pro tota ſerie; *vid. pag.* 176. Quadratur.

Sit, *e. g.* $e + fz^n + gz^{2n} + hz^{3n}$, *&c.* $= R$.

$k + lz^n + mz^{2n} + nz^{3n}$, *&c.* $= S$, *&c.* *&c.*

Erit $\overline{e + fz^n + gz^{2n}\ \&c.}|^{\lambda} = R^{\lambda}$ *&c.* *&c.* unde ſubſtituendo $RR^{\lambda-1} = R^{\lambda}$, *&c.* tum in operatione, tum in demonſtratione, & præcipue in directione ad ordinandas ſeries, quanto ſit uſui poſtea apparebit. Notationes aliorum quorundam (ſine ullo futuro fructu Novaturientium, ut nihil pejus dicam) hic omittam: Ejuſmodi ſunt quædam *Leibnitii*, quas hic memorare vix operæ pretium foret.

# CAP. III.

## *De Involutione & Evolutione ſpecioſâ.*

Inter cæteras *Newtoni* inventiones (quæ etiam methodo Fluxionum maxime ſubſerviunt) haud parum eminet illa quam primus omnium invexit pro involvenda quantitate quavis ſpecioſa, vel etiam pro eadem evolvenda, ſeu ad quaſvis ejuſdem radices extrahendas. Eandem (uti liquet e Cap. 91, 2*di* Vol. *Walliſii*) anno 1676, cum Regia Societate communicandam, in Epiſtolis duabus ad Dominum *Oldenburgium* ejuſdem, tum temporis, Secretarium, impertiverit, quæ quidem Epiſtolæ in tertio Vol. *Walliſii* extant, ut & ejuſdem rei mentio in ſecundo Vol. p. 369 *. Ibidem oſtendit *Uncias* (quas vocant) *ſeu numeros præfixos dignitatum membris a radice binomia procreatarum (quarum dignitatum exponentem vocat m) oriri ex hujuſmodi Multiplicatiene continua.*

$$1 \times \frac{m-0}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \text{ \&c.}$$

*qui quidem proceſſus, ſi m (exponens Poteſtatis) ſit numerus integer, ſeipſum determinabit poſt certum locorum numerum cuique poteſtati convenientem, redeunte unitate unde inceperat. Sed ſi m ſit Fractio, tranſibitur in infinitum per numeros negativos.* Quoad hujuſce autem Theorematis uſum cum omnibus hodie ſit familiaris, pluribus de eo dicendis ſuperſedemus. *Ex hoc* autem *proceſſu* (aliiſque conſimilis naturæ) hæcce Theoremata deducit pro hujuſmodi extractionibus, viz.

1°. $\overline{a+x}\big|^{m} = a^{m} + m\mathrm{AQ} + \frac{m-1}{2}\mathrm{BQ} + \frac{m-2}{3}\mathrm{CQ} + \frac{m-3}{4}\mathrm{DQ}$, &c. *vel*

2°. $\overline{\mathrm{P}+\mathrm{PQ}}\big|^{\frac{m}{n}} = \mathrm{P}^{\frac{m}{n}} + \frac{m}{n}\mathrm{AQ} + \frac{m-n}{2n}\mathrm{BQ} + \frac{m-2n}{3n}\mathrm{CQ} + \frac{m-3n}{4n}\mathrm{DQ}$ &c.

*ubi* P + PQ *eſt expoſita quantitas cujus Radix eſt extrahenda, aut aliqua poteſtas inde formanda :* P *eſt primus terminus ejuſmodi expoſitæ quantitatis ;* Q *eſt reſiduum ejuſdem (ſive reſiduum iſtud ſit monomium vel binomium vel trinomium, &c.) per* P *diviſum ; &* $\frac{m}{n}$ *exponens Radicis ſeu Dimenſionis quæſitæ.* Hoc autem Reſiduum ita intelligendum eſt (uti pag. 376. videre eſt) ut etiam *tum conſtruendis poteſtatibus, tum eliciendis radicibus quantitatis* plurimembris *inſerviat, quæ in ſeriem vel terminatam vel interminatum exeat, prout res tulerit.* Hujuſmodi proceſſus exempla octo (in Cap. 91.) addidit, e quibus duo vel tria hic tantum exſcribemus.

### Exemp. 1.

“ $\sqrt{cc+xx}$ ſeu $\overline{cc+xx}\big|^{\frac{1}{2}} = c + \frac{x^2}{2c} - \frac{x^4}{8c^3} + \frac{x^6}{16c^5} - \frac{x^8}{128c^7} + \frac{7x^{10}}{256c^9}$ &c.

“ nam in hoc caſu eſt $\mathrm{P} = cc$. $\mathrm{Q} = \frac{xx}{cc}$. $m = 1$. $n = 2$. $\mathrm{A} = (\mathrm{P}^{\frac{m}{n}} =$

“ $cc^{\frac{1}{2}}) = c$. $\mathrm{B}(= \frac{m}{n}\mathrm{AQ}) = \frac{xx}{cc}$. $\mathrm{C}(= \frac{m-n}{2n}\mathrm{BQ}) = \frac{-x^4}{8c^3}$ &c.

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* Anno ſcil. 64 & 65.

Exemp.

### Exemp. 2.

“ $\sqrt[5]{c^5+c^4-x^5}$ ſeu $\overline{c^5+c^4-x^5}|^{\frac{1}{5}} = c + \frac{c^4x-x^5}{5c^4}$
“ $-\frac{2c^8+4c^4x^6-2x^{10}}{25c^9}+$ &c. Quod manifeſtum erit, ſubſtitutis $1=m$,
“ $5=n$, $c^5=\mathrm{P}$. & $c^5$) $c^4x-x^5$ (Q. aut etiam ſubſtituamus $-x^5=\mathrm{P}$
“ & $-x^5$) $c^4x+c^5$ ( Q. Eritque $\sqrt[5]{c^5+c^4x-x^5} = -x+\frac{c^4x+c^5}{5x^4}$
“ $+\frac{2c^8x^2+4c^9x+c^{10}}{25x^9}+$ &c. Prior poſitio potior eſt, ſi ſit $x$ valde
“ parva, poſterior ſi valde magna.

### Exemp. 3.

“ $\frac{\mathrm{N}}{\sqrt[3]{y^3-aay}}$, hoc eſt $\mathrm{N}\times\overline{y^3-aay}|^{-\frac{1}{3}} = \mathrm{N}\times\frac{1}{y}+\frac{aa}{3y^3}+\frac{a^4}{3y^5}+\frac{7a^6}{81y^7}$ &c.
“ Nam hic $\mathrm{P}=y^3$. $\mathrm{Q}=\frac{-aa}{yy}$. $m=1$. $n=3$. A $(=\frac{m}{n}\mathrm{AQ}=-\frac{1}{3}\times\frac{1}{y}$
“ $\times\frac{-aa}{yy}) = \frac{aa}{3y^3}$ &c. &c. &c. Et per eandem Regulam poſſumus
“ tam in numeris, quam ſpeciebus, peragere generationem poteſtatum;
“ Diviſionem per poteſtates, aut per quantitates radicales; & extra-
“ ctionem radicum ſuperiorum poteſtatum, & ſimilia. Inter cæteras Theorematum horum inveſtigationes, libet hic unicam inſequentem quam mecum jam olim communicaverit D. *Heynes*, antequam aliorum, quæ extant, inveſtigationes publici juris factæ eſſent, quamque licet hoc loco anticipatam, hic apponere non hæremus. Inveſtigare ſcil. in ſerie quamvis radicem aut poteſtatem $a+x$ quæ exprimatur *v. g.* per $m$, *viz.* $\overline{a+x}|^m$.

Aſſumatur $\overline{a+x}|^m = b+cx+dx^2+ex^3$ &c.

Tunc erit $\overline{ma+x}|^{m-1}\dot{x} = c\dot{x}+2dx\dot{x}+3ex^2\dot{x}$ &c. & facta diviſione per $\dot{x}$.

erit $\overline{m\times\overline{a+x}|^{m-1}} = c + 2dx + 3ex^2$, &c.

Tum $\frac{m\times\overline{a+x}|^{m-1}}{\overline{a+x}|^{m}} = m\times\overline{a+x}|^{-1} = \frac{m}{a+x} =$

$\frac{c+2dx+3ex^2+4fx^3 \text{ \&c.}}{b+cx+dx^2+ex^3 \text{ \&c.}}$ & conſequenter $bm + cmx + dmx^2 + emx^3$ &c.

$\begin{array}{l} = ac + 2adx + 3aex^2 + 4afx^3 \text{ \&c.} \\ \qquad\quad cx + 2dx^2 + 3ex^3 \text{ \&c.} \end{array}$ & ponendo $b = a^m$, ex natura poteſtatum erit

$$\left.\begin{array}{l} c = ma^{m-1} \\ d = \frac{m-1}{2}\times ma^{m-2} \\ e = \frac{m-2}{3}\times\frac{m-1}{2}\times ma^{m-3} \end{array}\right\}\text{\&c.}$$ Comparando terminos homologos.

Et $\overline{a+x}|^m = a^m + ma^{m-1}x + \frac{m-1}{2}\times ma^{m-2}x^2 + \frac{m-2}{3}\times\frac{m-1}{2}\times ma^{m-3}x^3$ &c.

$= a^m + mA\frac{x}{a} + \frac{m-1}{2}B\frac{x}{a} + \frac{m-2}{3}C\frac{x}{a} +$ &c.

Tum ponendo A pro primo termino, B pro ſecundo, C pro tertio, *&c.*

& ponendo $\frac{a}{x} = Q$. erit $\overline{a+x}|^m = a^m + mAQ + \frac{m-1}{2}BQ + \frac{m-2}{3}CQ +$ *&c.* Quæ eſt ipſiſſima expreſſio D. *Newtoni* primo allata, &c. & æquatis æquandis ſecunda conſimili modo proveniet. Methodum autem ad elevandam actu quantitatem quamvis multinomiam ſeu infinitinomiam ad poteſtatem quamlibet indeterminatam vel etiam ad extractionem radicis cujuſlibet ex eadem, oſtendit Vir clariſſimus D. *A. de Moivre* in *Tranſact. Philoſ.* Num. 230, quam ab eodem in Linguam Latinam traductam hic apponemus; & notandum eſt indicem *m* vel numerum integrum eſſe, vel fractum, prout res exigit.

$$\overline{az + bz^2 + cz^3 + dz^4 + ez^5 \text{ \&c.}}|^m = a^m z^m + \frac{m}{1}a^{m-1}bz^{m+1} + \left.\begin{array}{l} \frac{m}{1}\times\frac{m-1}{2}a^{m-2}b^2 \\ \frac{m}{1}a^{m-1}c \end{array}\right\} x^{m+2}$$

$$+ \left.\begin{array}{l} \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}a^{m-3}b^3 \\ \frac{m}{1}\times\frac{m-1}{1}a^{m-2}bc \\ \frac{m}{1}a^{m-1}d \end{array}\right\} z^{m+3}$$

+

$$\left.\begin{array}{r} +\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\,a^{m-4}\,b^4 \\ \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{1}\,a^{m-3}\,b^2c \\ \frac{m}{1}\times\frac{m-1}{1}\,a^{m-2}\,bd \\ \frac{m}{1}\times\frac{m-1}{2}\,a^{m-2}\,c^2 \\ \frac{m}{1}\,a^{m-1}\,e \end{array}\right\}\, x^{m+4}\ \&c.\ \&c.$$

“ In hac ſerie, inquit, lex continuationis eſt hujuſmodi : Pro membris “ omnibus alicujus termini inveniendis, membra termini imme- “ diate præcedentis duco in $\frac{b}{a}$, item membra omnia termini hunc ul- “ timum immediate præcedentis duco in $\frac{c}{a}$, exceptis iis in quibus “ invenitur *b*. Iterum membra omnia termini hunc ultimum ſimili- “ ter præcedentis duco in $\frac{d}{a}$, exceptis iis in quibus invenitur *b* vel *c*. “ Rurſus membra omnia termini hunc ultimum immediate præcedentis “ duco in $\frac{e}{a}$, exceptis iis in quibus invenitur vel *b*, vel *c*, vel *d*. Et “ ſic per terminos omnes ante inventos regrediendo, invenientur, col- “ ligendo, membra omnia quæ iſtum ſeriei terminum ingredi poſſunt ; “ quorum unciæ, ſeu numeri præfigendi, ſic etiam reperiuntur. Pro “ quovis membro alicujus ſeriei termini, tot ſumantur termini ſeriei “ $m\times\overline{m-1}\times\overline{m-2}\times\overline{m-3}$ &c. quot ſunt unitates in ſumma indi- “ cum ipſarum *b*, *c*, *d*, *e*, &c. in eo membro ; fiat hæc numerator Fra- “ ctionis, cujus denominator erit productus ſerierum $1\times2\times3\times4$ &c. “ $1\times2\times3\times4$ &c. $1\times2\times3\times4\times5$ &c. quarum prima totidem ha- “ bet terminos quot ſunt unitates in indice ipſius *b*, ſecunda tot termi- “ nos habet quot ſunt unitates in indice ipſius *c*, tertia quot ſunt uni- “ tates in indice ipſius *d*, &c. Demonſtrationem, qui videre cupit, loco citato reperiet.

Hunc poro in finem D. *Cheynæus* methodum expeditiſſimam concinnavit, quæque (uti dicit pag. 58 *Meth. Flux. inverſ.*) *oculi intuitu continuatur in infinitum* ; *idque quidem facillime*, ſcilicet

$$\overline{a+bx+cx^2+dx^3+ex^4\ \&c.}\Big|^n = a^n + \frac{nbA}{a}x + \frac{2ncA+\overline{n-1}B}{2a}x^2$$

+

$$+\frac{3ndA+\overline{2n-1}cB+n-2bC}{3a}x^3+\frac{4neA+\overline{3n-1}dB+\overline{2n-2}cC+\overline{n-3}bD}{4a}x^4 \text{ \&c.}$$

*ubi*, inquit, *intuitu oculi natura progreſſionis ſe prodit*, v. g. *Decimus terminus eſt*

$$\frac{9nkA+\overline{8n-1}jB+\overline{7n-2}bC+\overline{6n-3}gD+\overline{3n-6}dG+\overline{2n-7}cH+n-8bj}{9a}x^9$$

*N. B.* In hiſce majuſcula quæq; denotat coefficientes totas immediate præcedentes ubi primum emicuerit, & eundem ubique obtinet valorem. Aliud exemplum adhuc complicatius addit, quod (cui animus ſit) conſulat Lector. Iis omnibus hocce juſtiſſimum Μνημόσυνον adjicit, *viz. Totum quod eſt hujus rei habebis ex Theoremate ipſius Newtoni inventoris, ſi enim (ipſo monente) pro primo quantitatis elevandæ termino, poſueris* p, *pro reliquis, primo excepto, poſueris* q, *elicies facilime omnia quæ in hiſce deſiderari poſſint.*

Facillime autem & maxime perſpicue, abſque labore obtinentur hujuſmodi termini, ſi modo tabulari diſponantur, *viz.*

Utravis

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $a^{n}$ | | | | | | | | | | |
| 2 | $nbAx$ | | | | | | | | | | |
| 3 | $2ncA$ | $n-1bBx^{2}$ | | | | | | | | | |
| 4 | $3ndA$ | $2n-1cB$ | $n-2Cx^{3}$ | | | | | | | | |
| 5 | $4neA$ | $3n-1dB$ | $2n-2cC$ | $n-3bDx^{4}$ | | | | | | | |
| 6 | $5nfA$ | $4n-1eB$ | $3n-2dC$ | $2n-3cD$ | $nb-4Ex^{5}$ | | | | | | |
| 7 | $6ngA$ | $5n-1fB$ | $4n-2eC$ | $3n-3dD$ | $2n-4cE$ | $n-5bFx^{6}$ | | | | | |
| 8 | $7nhA$ | $6n-1gB$ | $5n-2fC$ | $4n-3eD$ | $3n-4dE$ | $2n-5cF$ | $n-6bGx^{7}$ | | | | |
| 9 | $8niA$ | $7n-1hB$ | $6n-2gC$ | $5n-3fD$ | $4n-4eE$ | $3n-5dF$ | $2n-6cG$ | $n-7bHx^{8}$ | | | |
| 10 | $9nkA$ | $8n-1iB$ | $7n-2hC$ | $6n-3gD$ | $5n-4fE$ | $4n-5eF$ | $3n-6dG$ | $2n-7cH$ | $n-8bIx^{9}$ | | |
| 11 | $10nlA$ | $9n-1kB$ | $8n-2iC$ | $7n-3hD$ | $6n-4gE$ | $5n-5fF$ | $4n-6eG$ | $3n-7dH$ | $2n-8cI$ | $n-9bKx^{10}$ | |
| 12 | $11nmA$ | $10n-1lB$ | $9n-2kC$ | $8n-3iD$ | $7n-4hE$ | $6n-5gE$ | $5n-6fG$ | $4n-7eH$ | $3n-8dI$ | $2n-9cK$ | $n-10bLx^{11}$<br>&c. ad inf |

Et unumquodque hujus Seriei membrum dividitur, nempe primum per $a$, ſecundum per $2a$, tertium per $3a$, & ſic porro ad inf. *h.e.* Series ſupra expoſita Numeratores, & altera Series, viz. $a$, $2a$, $3a$, &c. denominatores conſtituunt ad inf.

*Hujuſce Theorematis Inveſtigationem in ſuo loco exhibebimus.*

Utravis methodo poteſtates & radices pro quantitatibus trinomiis, quadrinomiis, &c. obtineri poſſe, ponendo ſcil. coefficientes omnes poſt ultimum nomii particularis quæſiti æquales nihilo, &c. Evidentius patet quam ut eidem explicandæ immorari hic opus ſit; adeo ut revera nihil in hiſce jam ultra deſiderari poſſe videtur, quod non facillime obtineri queat.

# CAP. IV.

## *De Fluxionum Methodo directâ.*

ANNO 1687, Vir celeberrimus *Iſ. Newtonus* librum ſuum incomparabilem, cui titulus *Principia Philoſophiæ Naturalis Mathematica*, evulgavit; qui magna ex parte hujuſmodi calculo debetur, uti rectiſſime obſervavit illuſtriſſimus *Marchio Hoſpitallius* hiſce verbis in præfatione ad tractatum ſuum hocce inſignitum titulo, *L'Analyſe des Infiniment Petits*, in quo cum cæteris quibuſdam *Germaniæ* & *Galliæ* Mathematicis perſuaſum ſibi habuerit, *Leibnitium* hujuſce calculi primum extitiſſe authorem, veritatem tamen ſubinde ſuſpicatus, in hæcce verba prorumpit, *Il eſt vray que le ſcavant Monſieur* Newton *a trouvé quelque choſe de ſemblable*, &c. *comme il paroit par ſon excellent Livre intitulé* Principia, &c. *qui eſt preſque tout de ce calcul.* In hocce, inquam, libro calculi hujuſce ſui Algorithmum primò publico impertiverit p. 256. cujus innumera fere atque egregia exempla ibidem oſtendit, & dudum antea multis per *Europam* Mathematicis ejuſdem copiam conceſſerat; notatione autem diverſa ab iſta qua alias uſus eſt, in demonſtratione ibidem allata, rem peragit; Sed & eundem notatione ſua maxime uſitata cum *Wal-*

*liſio* poſtea communicavit, uti pag. 292, vol. 2, conſtat, quemque tractatui ſuo de Quadratura curvarum inſeruit propoſitione hacce problematica comprehenſum, *viz. Data æquatione quotcunque fluentes quantitates involvente invenire Fluxiones.*

## SOLUTIO.

*Multiplicetur omnis æquationis terminus per indicem dignitatis quantitatis cujuſque fluentis quam involvit, & ſingulis Multiplicationibus mutetur dignitatis latus in Fluxionem ſuam, & aggregatum factorum omnium ſub propriis ſignis erit æquatio nova, quæ relationem Fluxionum involvet.*

## EXPLICATIO.

*Sunto* a, b, c, d, *&c. quantitates determinatæ & immutabiles, & proponatur æquatio quævis quantitates fluentes* z, y, x, *&c. Involvens uti* $x^3 - xyy + aaz - b^3 = 0$: *Multiplicentur termini primo per indices dignitatum* x, *& in ſingulis Multiplicationibus pro dignitatis latere, ſeu* x *unius dimenſionis ſcribatur* $\dot{x}$, *& ſumma factorum erit* $3\dot{x}x^2 - \dot{x}yy$. *Idem fiat in* y, *& prodibit* $-2xy\dot{y}$. *Idem fiat in* z, *& prodibit* $aa\dot{z}$. *Ponatur ſumma factorum æqualis nihilo, & habebitur æquatio* $3\dot{x}x^2 - \dot{x}yy - 2xy\dot{y} + aa\dot{z} = 0$. *Dico quod hac æquatione definitur relatio Fluxionum.*

## DEMONSTRATIO.

*Nam ſit* o *quantitas admodum parva & ſunto* $o\dot{z}$, $o\dot{y}$, $o\dot{x}$, *quantitatum* z, y, x, *momenta, id eſt, incrementa momentanea ſynchrona. Et ſi quantitates fluentes jam ſunt* z, y, x, *hæ poſt momentum temporis incrementis ſuis* $o\dot{z}$, $o\dot{y}$, $o\dot{x}$, *auctæ evadent* $z + o\dot{z}$, $y + o\dot{y}$, $x + o\dot{x}$, *quæ in æquatione prima pro* z, y, *&* x, *ſcriptæ dant æquationem* $x^3 + 3xxo\dot{x} + 3xoo\dot{x}\dot{x} + o^3\dot{x}^3 + xyy - o\dot{x}yy - 2xoy\dot{y} - 2\dot{x}ooy\dot{y} - xoo\dot{y}\dot{y} - \dot{x}o^3\dot{y}\dot{y} + aax + aao\dot{z} - b^3 = 0$. *Subducatur æquatio prior, & reſiduum diviſum per* o *erit* $3\dot{x}x^2 + 3\dot{x}\dot{x}ox + \dot{x}^3oo - \dot{x}yy - 2xy\dot{y} - 2\dot{x}oy\dot{y} - xo\dot{y}\dot{y} + aax = 0$. *Minuatur quantitas* o *in infinitum, & neglectis terminis evaneſcentibus reſtabit* $3\dot{x}x^2 - \dot{x}yy - 2xy\dot{y} + aa\dot{z} = 0$. *Q. E. D.*

## *Explicatio Plenior.*

*Ad eundem modum ſi æquatio eſſet* $x^3 - xyy + aa\sqrt{ax-yy-b^3} = 0$, *produceretur* $3x^2\dot{x} - \dot{x}yy - 2x\dot{y}y + aa\dot{\sqrt{ax-yy}} = 0$. *Ubi ſi Fluxionem* $\dot{\sqrt{ax-yy}}$ *tollere veliis, pone* $\sqrt{ax-yy} = z$, *& erit* $ax - yy = z^2$, *& (per hanc propoſitionem)* $a\dot{x} - 2y\dot{y} = 2z\dot{z}$ *ſeu* $\frac{a\dot{x} - 2y\dot{y}}{2z} = \dot{z}$, *hoc eſt* $\frac{a\dot{x} - 2y\dot{y}}{2\sqrt{ax-yy}} = \dot{\sqrt{ax-yy}}$. *& inde* $3x^2\dot{x} - \dot{x}yy - 2x\dot{y}y + \frac{a^3\dot{x} - 2aay\dot{y}}{2\sqrt{ax-yy}} = 0$. *& per operationem repetitam pergitur ad Fluxiones ſecundas, tertias, & ſequentes. Ad eundem modum ſi æquatio eſſet* $x^3 - xyy + aa\sqrt{ax-yy}-b^3 = 0$. *Ubi ſi Fluxionem* $\dot{\sqrt{ax-yy}}$ *tollere velis, pone* $\sqrt{ax-yy} = z$, *& erit* $ax - yy = z^2$: *Et (per hanc propoſitionem)* $a\dot{x} - 2y\dot{y} = 2z\dot{z}$ *ſeu* $\frac{a\dot{x} - 2y\dot{y}}{2z} = \dot{z}$, *hoc eſt* $\frac{a\dot{x} - 2y\dot{y}}{2\sqrt{ax-yy}} = \dot{\sqrt{ax-yy}}$ *& inde* $3x^2\overset{:}{x} - \dot{x}yy - 2x\dot{y} + \frac{a\dot{x} - 2y\dot{y}}{2\sqrt{ax-yy}} = 0$. *& per operationem repetitam pergitur ad Fluxiones ſecundas, tertias, & ſequentes.*

*Sit æquatio* $zy^3 - z^4 + a^4 = 0$, *& provenient per operationem primam*

$$\begin{array}{r} \text{Flux. primæ.} \\ \text{Inde Flux. ſecundæ.} \\ \\ \\ \end{array} \left\{ \begin{array}{c|c|l} \dot{z}y^3 & +3z\dot{y}y^2 & -4\dot{z}z^3 = 0. \\ \hline \ddot{z}y^3 & 3\dot{z}\dot{y}y^2 & -4\ddot{z}z^3 \\ 3\dot{z}\dot{y}y^2 & 6z\dot{y}^2y & -12\dot{z}^2z^2 \\ & 3z\ddot{y}y^2 & \end{array} \right.$$

Flux.

| | | | | | | |
|---|---|---|---|---|---|---|
| Summa | $\ddot{z}y^3 +$ | $6\dot{z}\dot{y}y^2$ | $+ 6z\dot{y}^2y +$ | $3z\ddot{y}y^2$ | $- 4\ddot{z}z^2 -$ | $12\dot{z}^2z^2$ |
| Flux. tertiæ. | $\dddot{z}y^3 +$ | $6\ddot{z}\dot{y}y^2$ | $+ 6\dot{z}\dot{y}^2y$ | $3\dot{z}\ddot{y}y^2$ | $- 4\dddot{z}z^3$ | $24\dot{z}\ddot{z}z^2$ |
| | $3\ddot{z}\dot{y}y^2$ | $12\dot{z}\dot{y}^2y$ | $+ 12z\dot{y}\ddot{y}y$ | $6z\ddot{y}\dot{y}y$ | $- 12\ddot{z}\dot{z}z^2$ | $24\dot{z}^3z$ |
| | | $6\dot{z}\ddot{y}y$ | $6z\dot{y}^3$ | $3z\dddot{y}y^2$ | | |

$$\dddot{z}y^3 + 9\ddot{z}\dot{y}y^2y + 18\dot{z}\dot{y}^2 + 3z\dddot{y}y^2 + 18z\dot{y}\ddot{y}y + 18\dot{z}\ddot{y}yy + 6z\dot{y}^3 + 9\dot{z}\ddot{y}y^2$$
$$- 4\dot{z}\ddot{z}^3 - 36\ddot{z}\dot{z}z^2 - 24\dot{z}^3z.$$

*Ubi vero pergitur ad Fluxiones ſecundas, tertias, & ſequentes, convenit quantitatem aliquam ut uniformiter fluentem conſiderare, & pro ejus Fluxione prima unitatem ſcribere, pro ſecunda vero & ſequentibus nihil. Sit æquatio*

$$\dot{z}y^3 - z^4 + a^4 = 0,$$

*& fluat* $z$ *uniformiter, ſitque ejus Fluxio unitas, & fiet per operationem primam* $y^3 + 3z\dot{y}y^2 - 4z^3 = 0$; *per ſecundam* $6\dot{y}y^2 + 3z\ddot{y}y^2 + 6z\dot{y}^2y - 12z^2 = 0$; *per tertiam* $9\ddot{y}y^2 + 18\ddot{y}^2y + 3z\ddot{y}y^2 + 18z\dot{y}\ddot{y}y + 6z\dot{y}^3 - 24z = 0$.

*In hujus autem generis æquationibus concipiendum eſt quod Fluxiones in ſingulis terminis ſint ejuſdem ordinis, id eſt vel omnes primi ordinis* $\dot{y}$, $\dot{z}$, *vel omnes ſecundi* $\ddot{y}$, $\dot{y}^2$, $\dot{y}\dot{z}$, $\dot{z}^2$, *vel omnes tertii* $\dddot{y}$, $\ddot{y}\dot{y}$, $\ddot{y}\dot{z}$, $\dot{y}^3$, $\dot{y}^2\dot{z}$, $\dot{y}\dot{z}^2$, $\dot{z}^3$, &c. *& ubi res aliter ſe habet complendus eſt ordo per ſubintellectas Fluxiones quantitatis uniformiter fluentis. Sic æquatio noviſſima complendo ordinem tertiam fit*

$$9\dot{z}\ddot{y}y^2 + 18\dot{z}\dot{y}^2y + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3 - 24z\dot{z}^3 = 0.$$

Et iiſdem methodis (uti ſupra pro quantitatibus radicalibus) inveniuntur Fluxiones quantitatum quarumvis fractarum, viz. Si propoſitum eſſet invenire Fluxionem quantitatis $\frac{a}{x}$ vel $\frac{x}{y}$, ponendo 1° $\frac{a}{x} = z$, habebitur pro Fluxione $\frac{a\dot{x}}{xx} = \dot{z}$ & pro 2° $\frac{y\dot{x} - x\dot{y}}{yy} = \dot{z}$, & ſic pro cæteris.

Jam vero pro Fluxionibus quantitatum Logarithmicarum † $\varphi : lx = \frac{\dot{x}}{x}$ & $\varphi\,\overline{lx + 1} = \frac{\dot{x}}{x+1}$ &c. Sed de hiſce ſpecialius in Cap. 7. agendum, quapropter pluribus de iiſdem in hoc loco dicendis ſuperſedemus. Sed & etiam $\varphi : R^{\lambda} = \lambda\dot{R}R^{\lambda-1}$ ubi ſi $\lambda$ ſit fractio, in indicem negativum tranſit, h. e. uti ſupra notavimus, quantitatem, cui præfixa, eſt in diviſorem vertit. Sic v. g. invenire Fluxionem $az^{\theta} \times \overline{e + fz^{n}}|^{\lambda} = QR^{\lambda}$, Invenietur $\dot{Q}RR^{\lambda-1} + \lambda Q\dot{R}R^{\lambda-1}$, hoc eſt $\overline{\dot{Q}R + \lambda Q\dot{R}}| \times R^{\lambda-1}$ quæ facillime in terminis explicitis invenitur; ubi ſi $\lambda$ unitate minor eſt $R^{\lambda-1}$ in diviſorem abit, e. g. Detur $\overline{e + fz^{n}}|^{\frac{1}{2}}$, erit ejuſdem Fluxio $nf\dot{z}z^{n-1} \times 2 \times \overline{e+fz^{n}}|^{-\frac{1}{2}} = \frac{nf\dot{z}z^{n-1}}{2\sqrt{e+fz^{n}}}$ * & ſic pro cæteris.

Methodo porro directa ex datis arearum expreſſione analytica, vel etiam contento ſolidi geniti rotatione plani circa axem curvilinei, obtinentur curvarum ordinatæ, pariter ac natura ſeu relatio coordinatarum, Tangentes & innumera alia, quæ hic recenſere nimis eſſet longum.

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† $\varphi$: *denotat Fluxionem quantiatis cujuſcunque*, & F: *Fluxionis cujuſlibet Fluentem.*

* *Exempla alia in ſequentibus invenire eſt.*

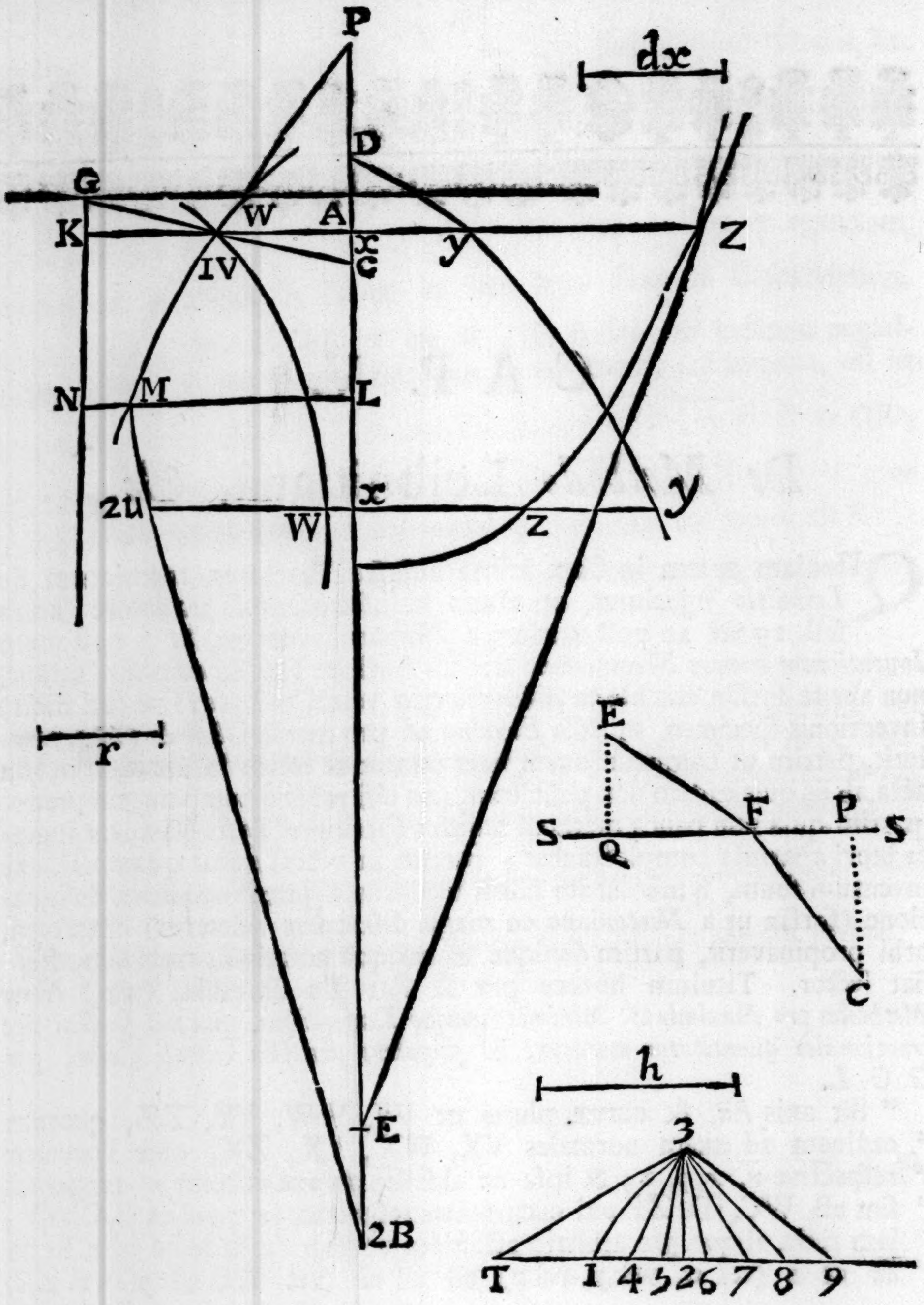
# CAP. V.

## *De Methodo* Leibnitianâ, *&c.*

QUoniam autem in Cap. primo hujuſce Tractatus, mentionem de *Leibnitio* injecimus, uti altero hujuſce methodi inventore (annis ſcil. 19 vel 20 poſt eandem a *Neutono* inventam, & 8 poſtquam *Inventionem hancce Neutonianam* ſibi ab Authore habuit communicatam) non abs re forſan erit hic ea adducere quæ veluti primum ſuæ (uti dicit) Inventionis ſpecimen, in *Actis Eruditorum* pro menſe *Octobris* 1684, attulerit, partim ut comparationem inter utramque lector inſtituat, cum iſta acta, ab iis quæ eadem non poſſideant, non niſi magno comparantur pretio; partim quia non pauca methodi hujuſce ſpecimina Tyronibus non ingrata brevi chartula complectantur; partim ut videri queat quanto labore inventum ſuum, ſymbolorum nimis inconcinna præ *Neutoniana* deſignatione (forſan ut a *Neutoniana* eo magis diſcrepare videretur) obtectum, orbi propinaverit, partim denique ut utrique notationis methodo aſſuefiat lector. Titulum huncce præ ſe fert iſta chartula, (viz.) *Nova Methodus pro Maximis & Minimis itemque Tangentibus, quæ nec fractas nec irrationales quantitates moratur, & ſingulare pro illis calculi genus, per G. G. L.*

“ Sit axis $Ax$, & curvæ plures ut VV, WW, YY, ZZ, quarum
“ ordinatæ ad axem normales VX, WX, YX, ZX, quæ vocentur
“ reſpective $v$, $w$, $y$, $z$; & ipſa $ax$ abſciſſa ab axe vocetur $x$, tangentes
“ ſint $v$B, WC, $y$D, ZE axi occurrentes reſpective in punctis B, C, D, E;
“ jam recta aliqua pro arbitrio aſſumpta vocetur $dx$, & recta quæ ſit ad
“ $dx$ ut $u$ (vel $w$, vel $y$, vel $z$) eſt ad $vb$ (vel WC ad $y$D vel ZE)

“ vocetur


P
dx
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“ vocetur $dv$ (vel $dw$, vel $dy$, vel $dz$) ſive differentia ipſarum $v$ (vel ip-
“ ſarum $w$, aut $y$, aut $z$) his poſitis calculi regulæ erunt tales.

“ Sit $a$ quantitas data conſtans, erit $da = o$, & $dax = adx$ : ſi ſit
“ $y = v$ (ſeu ordinata quævis curvæ $yy$ æqualis cuivis ordinatæ reſpon-
“ denti curvæ $vv$ (erit $dy = dv$) jam additio & ſubſtractio, ſi ſit $z - y$
“ $+ vv + x = v$, erit $d\overline{z - y + vv + x}$, ſeu $dv = dz - dy + dvv$
“ $+ dx$. Multiplicatio, $d : xv = xdv + vdx$, ſeu poſito $y = xv$ fiet
“ $dy = xdv + vdx$ : in arbitrio enim eſt vel formulam ut $xv$, vel com-
“ pendio pro ea literam ut $y$ adhibere. Notandum & $x$ & $dx$ eodem
“ modo in hoc calculo tractari ut $y$ & $dy$, vel aliam literam indetermina-
“ tam cum ſua differentiali ; notandum etiam non dari ſemper regreſ-
“ ſum a differentiali æquatione niſi cum quadam cautione, de quo
“ alibi. Porro diviſio $d\frac{v}{y}$ vel (poſito $z = \frac{v}{y}$) $dz = \frac{vdy \pm ydv}{yy}$ quoad ſi-
“ gna hoc probe notandum, cum in calculo pro litera ſubſtituitur ſim-
“ pliciter ejus differentialis, ſervari quidem eadem ſigna, & pro $+ z$
“ ſcribi $+ dz$, pro $- z$ ſcribi $- dz$, ut ex additione & ſubſtractione
“ paulo ante poſita apparet : Sed quando ad Exegeſin valorum venitur,
“ ſeu cum conſideratur ipſius $z$ relatio ad $x$, tum apparere an valor ip-
“ ſius $dz$ ſit quantitas affirmativa an nihilo minor ſeu negativa, quod
“ poſterius cum ſit, tum tangens $z$E ducitur a puncto $z$ non verſus A
“ ſed in partes contrarias, ſeu infra $x$, id eſt tunc cum ipſæ ordinatæ $z$
“ decreſcunt creſcentibus $x$, & quia ipſæ ordinatæ $v$ modo creſcunt
“ modo decreſcunt, erit $dv$ modo affirmativa modo negativa quantitas,
“ & priore caſu $V_1B$ tangens ducitur verſus A, poſteriore $2V_2B$ in
“ partes averſas : neutrum autem fit in medio circum M quo mo-
“ mento ipſæ $v$ neque creſcunt neque decreſcunt ſed in ſtatu ſunt,
“ adeoque fit $dv = o$, ubi nihil refert quantitas ſitne affirmativa an ne-
“ gativa ; nam $+ o = - o$ ; eoque in loco ipſa $v$, nempe ordinata $l$M
“ eſt maxima (vel ſi convexitatem axi obverteret minima) & Tangens
“ curvæ in M, neque ſupra X ducitur ad partes A, ibique axi appro-
“ pinquat, neque infra X ad partes contrarias, ſed eſt axi parallela.
“ Si $dv$ ſit infinita reſpectu ipſius $dx$, tum tangens eſt ad axem recta,
“ ſeu eſt ipſa ordinata. Si $dv$ & $dx$ æquales, tangens facit angulum
“ ſemirectum ad axem. Si creſcentibus ordinatis $v$, creſcunt etiam ipſa
“ earum incrementa vel differentiæ $dv$ (ſeu ſi poſitis $dv$ affirmativis,
“ etiam $ddv$ differentiæ differentiarum ſunt affirmativæ, vel negativis
“ negativæ) curva axi obvertit concavitatem, alias convexitatem :
“ ubi vero eſt maximum vel minimum incrementum, vel ubi incrementa
“ ex decreſcentibus fiant creſcentia aut contra, ibi eſt punctum flexus con-
“ trarii, & concavitas & convexitas inter ſe permutantur, modo non &

“ ordinatæ ibi ex creſcentibus fiant decreſcentes vel contra, tunc enim
“ concavitas aut convexitas maneret, aut incrementa continuent creſcere
“ aut decreſcere, ordinatæ vero ex creſcentibus fiant decreſcentes,
“ vel contra fieri non poteſt. Itaque punctum flexus contrarii locum
“ habet quando neque $v$, neque $dv$ exiſtente o, tamen $ddv$ eſt o. Unde
“ etiam Problema flexus contrarii non duas ut problema maximæ, ſed
“ tres habet radices æquales, atque hæc omnia quidem pendent a recto
“ uſu ſignorum. Interdum autem adhibenda ſunt ſigna ambigua ut
“ nuper in diviſione, antequam ſcilicet conſtet quomodo explicari
“ debeant, & quidem ſi creſcentibus $x$, creſcunt (decreſcunt) $\frac{v}{y}$, debent
“ ſigna ambigua in $d\frac{v}{y}$ ſeu in $\frac{vdy \mp ydv}{yy}$ ita explicari ut hæc fractio fiat
“ quantitas affirmativa (negativa) ſignificat autem $\mp$ contrarium ipſius
“ $+$, ut ſi hoc ſit $+$ illud ſit $-$, vel contra, poſſunt & in eodem calculo
“ occurrere plures ambiguitates quas diſtinguo parentheſibus,
“ exempli cauſa, ſi eſſet $\frac{v}{y} + \frac{y}{z} + \frac{x}{v} = w$ foret $\frac{\pm vdy \mp ydv}{yy} + \frac{(\pm)\, ydz (\mp)\, zdy}{zz}$
“ $+ \frac{(\mp) xdv (\pm) vdx}{vv} = dw$; alioqui ambiguitates ex diverſis capitibus ortæ
“ confunderentur, ubi notandum ſignum ambiguum in ſeipſum ductum
“ dare $+$, in ſuum contrarium dare $-$; in aliud ambiguum formare
“ novam ambiguitatem ex duabus dependentem.

“ Potentiæ $dx^a = ax^{a-1}\,dx$, exempli gratia $\overline{dx^3} = 3x^2\,dx$, $d, \frac{1}{x^a} = \frac{adx}{x^{a+1}}$
“ ſi $w = \frac{1}{x^3}$ fiet $dw = -\frac{3dx}{x^4}$; radices $d\sqrt[b]{x^a} = \frac{a}{b}\,dx\,\sqrt[b]{x-b}$ (hinc
“ $d\sqrt[2]{y} = \frac{dy}{2\sqrt[2]{y}}$ nam eo caſu $a$ eſt (1) & $b$ (2) ergo $\frac{a}{b}\,dx\,\sqrt[b]{x^{a-b}}$ eſt
“ $\frac{1}{2}\sqrt[2]{y-1}$, jam $y^{-1}$ idem eſt quod $\frac{1}{y}$ ex natura exponentium progreſſionis
“ geometricæ, & $\sqrt[2]{\frac{1}{y}}$ eſt $\sqrt[\frac{1}{2}]{y}$ & $d\frac{1}{\sqrt[b]{x^a}} = -\frac{bdx}{a\sqrt[b]{x^{b-a}}}$;
“ ſuffeciſſet autem regula potentiæ integræ tam ad fractas quam ad
“ radices determinandas, potentia enim fit fracta cum exponens eſt
“ negativus, & mutatur in radicem cum exponens eſt fractus, ſed
“ malui conſequentias ipſas deducere, quam aliis deducendas relinquere,
“ cum ſint admodum generales & crebro occurreentes & in re
“ per ſe implicita præſtat facilitati conſulere. Ex cognito hoc velut
“ algorithmo, ut ita dicam, calculi hujus, quem voco differentialem,
omnes

“ omnes aliæ æquationes differentiales inveniri poſſunt per calculum communem, maximæque & minimæ itemque tangentes haberi, ita ut opus non ſit tolli fractas aut irrationales, aut alia vincula quod tamen faciendum fuit ſecundum methodos hactenus editas, demonſtratio omnium facilis erit in hiſce rebus verſato & hoc unum hactenus non ſatis expenſum conſideranti ipſas $dx$, $dy$, $dv$, $dw$, $dz$, ut ipſarum $x$, $y$, $v$, $w$, $z$, (cujuſque in ſua ſerie) differentiis ſive incrementis vel decrementis momentaneis proportionales haberi poſſe, unde fit ut propoſita quacunque æquatione ſcribi poſſit ejus æquatio differentialis quod *fit* pro quolibet membro (id eſt parte quæ ſola additione vel ſubſtractione ad æquationem conſtituendam concurrit) ſubſtituendo ſimpliciter quantitatem membri differentialem, pro alia vero quantitate (quæ non ipſa eſt membrum ſed ad membrum formandum concurrit) ejus quantitatem differentialem ad formandam quantitatem differentialem ipſius membri adhibendo; non quidem ſimpliciter ſed ſecundum Algorithmum hactenus præſcriptum, * editæ vero hactenus methodi talem tranſitum non habent, adhibent enim plerumque rectam ut DX vel aliam hujuſmodi, non vero rectam $dy$ quæ ipſis $Dx$, $xy$, $dx$ eſt quarta proportionalis, quod omnia turbat. Hinc præcipiunt ut fractæ & irrationales (quas indeterminatæ ingrediuntur) prius tollantur. Patet etiam methodum noſtram porrigi ad *lineas tranſcendentes, quæ ad calculum Algebraicum revocari non poſſunt, ſeu quæ nullius ſunt certi gradus*, idque univerſaliſſimo modo ſine ullis ſuppoſitionibus particularibus non ſemper ſuccedentibus, modo teneatur in genere, Tangentem invenire, eſſe rectam ducere quæ duo curvæ puncta diſtantiam infinite parvam habentia jungat ſeu latus productum Polygoni infinitanguli, quod nobis curvæ æquivalet; diſtantia autem illa infinite parva ſemper per aliquam differentialem notam, ut $dv$, vel per relationem ad ipſam exprimi poteſt, hoc eſt per notam quandam Tangentem; ſpeciatim ſi eſſet $y$ quantitas tranſcendens, exempli cauſa, ordinata Cycloidis, eaque calculum ingrederetur cujus ope ipſa $z$ ordinata alterius curvæ eſſet determinata, & quæreretur $dz$ ſeu per eam Tangens hujns curvæ poſterioris, utique determinanda eſſet $dz$ per $dy$, haberetur autem $dy$ quia habetur tangens Cycloidis, ipſa autem tangens Cycloidis ſi nondum haberi fingeretur, ſimiliter calculo inveniri poſſit ex data proprietate Tangentium circuli. Placet autem exemplum calculi †, ubi notetur

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* *Hic loquitur ac ſi conſcius eſſet ineditæ, hujuſce generis, methodi, ſcil.* Neutonianæ.

† [Credo] *adferre.*

“ me diviſionem hic deſignare hoc modo $x : y$, quod idem eſt ac $x$ diviſ.
“ per $y$, ſeu $\frac{x}{y}$; Sit æquatio prima ſeu data $x : \overline{y+a} + b \times \overline{c-xx}$ quadr.
“ $\frac{y}{ex+fxx} + ax\sqrt{\overline{gg+yy+yy}} : \sqrt{\overline{bb+lx+mxx}} = 0$. Exprimens relationem
“ inter $x$ & $y$, ſeu inter A$x$ & $xy$, poſito ipſas $a$, $b$, $c$, $d$, $e$, $f$,
“ $g$, $b$, $l$, $m$, eſſe datas, quæritur modus ex dato puncto $y$ educendi
“ YD quæ curvam tangat ſeu quæritur ratio rectæ DX ad rectam datam
“ XY; compendii cauſa pro $a + bx$ ſcribamus $n$, pro $c - xx$, $p$;
“ pro $ex + fxx$, $q$; pro $gg + yy$, $r$; pro $bb + lx + mxx$, $s$; fiet
“ $x : y + np : qq + ax\sqrt{\overline{r+yy}} : \sqrt{s} = 0$. Quæ ſit æquatio ſecunda, jam
“ ex calculo noſtro $dx : y$ eſſe $+ xdy + ydx : yy$ & ſimiliter $d, np : qq$
“ eſſe $(\pm)\ 2npdq\ (\mp)\ qnd\ \overline{+pdn} : q^3$ & $dax\sqrt{r}$ eſſe $- axdr : 2r\sqrt{}\,\overline{+adx}\sqrt{r}$;
“ & $dyy : \sqrt{as}$ eſſe $((\pm))\ yyds\ ((\mp)) - 4ysdy : 2s\sqrt{s}$ quæ omnes quantitates
“ differentiales inde ab ipſo $dx : y$ uſque ad $d, yy : \sqrt{s}$ in unum
“ addita faciunt 0 & dabunt hoc modo æquationem tertiam, ita enim
“ pro membris ſecundæ æquationis ſubſtituuntur quantitates eorum differentiales.
“ Jam $dn$ eſt $bdx$ & $dp$ eſt $- 2xdx$, & $dq$ eſt $edx + 2fxdx$,
“ & $dr$ eſt $2ydy$ & $ds$ eſt $ldx + 2mxdx$, quibus valoribus in æquatione
“ tertia ſubſtitutis habebitur æquatio quarta, ubi quantitates differentiales
“ quæ ſolæ ſuperſunt, nempe $dx$, $dy$ ſemper reperiuntur extra nominatores
“ & vincula, & unumquodque membrum afficitur vel per $dx$,
“ vel per $dy$ ſervata ſemper lege homogeneorum, quoad has duas quantitates,
“ quomodocunque implicatus ſit calculus, unde ſemper haberi
“ poteſt valor ipſius $dx : dy$, ſeu rationis $dx$ ad $dy$, hoc eſt DX quæſitæ
“ ad XY datam, quæ ratio in hoc noſtro calculo (mutando æquationem
“ quartam in analogiam) erit ut $\pm x : y + a \times y : \sqrt{r}\ ((+))\ 2y : \sqrt{s}$
“ eſt ad $\mp 1 : y\ (+)\ 2npe + 2fx : q^3\ (+) - 2n \times + pb : qq + a\sqrt{r}$
“ $((+))\ yyl + 2mx : 2s\sqrt{s}$: dantur autem $x$ & $y$ ex dato puncto $y$, dantur
“ & valores ſupraſcripti literarum $n$, $p$, $q$, $r$, $s$, per $x$ & $y$, habetur ergo
“ quæſitum, atque hoc exemplum ſatis implicatum ideo tantum adſcripſimus,
“ ut modus ſuperioribus regulis in calculo etiam difficiliore
“ utendi appareret; nunc præſtat uſum in exemplis intellectui magis
“ obviis oſtendere. Data ſint duo puncta C & E, & recta SS in eodem
“ cum ipſis plano, quæritur punctum F in recta SS ita ſumendum ut junctis
“ CF, FE ſit aggregatum rectangulorum CF in datam $nh$ & FE in
“ datum $r$ omnium poſſibilium minimum hoc, eſt ſi, SS ſit mediorum
“ ſeparatrix, & $h$ repreſentet denſitatem medii, ut aquæ a parte C, & $r$
“ denſitatem medii ut aeris a parte E, quæritur punctum F tale ut via
“ a C ad E per F ſit omnium poſſibilium facillima, ponamus omnia

“ iſta rectangulorum aggregata poſſibilia, vel omnes viarum poſſibilium
“ difficultates repræſentari per ipſas KV, curvæ VV ordinatas ad rectam
“ GK normales, quas vocabimus $\omega$, quærique minimam earum
“ NM, quia dantur puncta C & E dabuntur & perpendiculares ad
“ SS nempe CP (quam vocabimus $c$) & EQ (quam $e$) & præterea PQ
“ (quam $p$) ipſam autem QF quæ ſit æqualis ipſi GN (vel AX) vocabimus
“ $x$, & CF, $f$; & EF, $g$; fiet FP, $p-x$; $f=\sqrt{cc+pp-2px+xx}$,
“ ſeu compendio $\sqrt{l}$, & $g=\sqrt{ee+xx}$, ſeu compendio $\sqrt{m}$, habemus
“ ergo $\omega=b\sqrt{l}+r\sqrt{m}$, cujus æquationis æquatio differentialis poſito
“ $d\omega$ eſſe $0$ in caſu minimæ) eſt $0=-bdl:\sqrt[2]{l}-rdm:2\sqrt{m}$ per regulas
“ calculi noſtri traditas; jam $dl$ eſt $-2dx\,\overline{-p+x}$, & $dm$ eſt $2xdx$,
“ ergo fit $b\overline{p-x}:f=rx:g$; quod ſi jam hæc accommodentur ad
“ Dioptricam, & ponantur $f$ & $g$, ſeu CF & EF æquales, quia eadem
“ manet refractio in puncto F quantacunque ponatur longitudo rectæ
“ CF, fiet $b\overline{p-x}=rx$, ſeu $b:r::x:p-x$, ſeu $b$ ad $r$ ut QF ad
“ FP, hoc eſt ſinus angulorum incidentiæ & refractionis FP & QF erunt
“ reciproce ut $r$ & $b$ denſitates mediorum in quibus fit incidentia & refractio.
“ Quæ denſitas tamen non reſpectu noſtri, ſed reſpectu reſiſtentiæ
“ quam radii lucis faciunt, intelligenda eſt. Et habetur ita demonſtratio
“ calculi, alibi a nobis in his actis exhibiti, quando generale Opticæ,
“ Catoptricæ, & Dioptricæ fundamentum exponebamus. Cum alii doctiſſimi
“ Viri multis ambagibus venati ſint quæ hujus calculi peritus tribus
“ lineis impoſterum præſtabit, quod alio adhuc exemplo docebo. Sit
“ curva 133 talis naturæ, ut a puncto ejus quocunque ut 3 ductæ ad ſex
“ puncta fixa in axe poſita, 4, 5, 6, 7, 8, 9, ſex rectæ 34, 35, 36, 37, 38,
“ 39 ſimul additæ, ſint rectæ datæ $g$, æquales; Sit axis T14526789,
“ & 12 ſit abſciſſa, 23 ordinata, quæritur Tangens 3 T dico fore T2 ad 23
“ ut $\frac{23}{34}+\frac{23}{35}+\frac{23}{36}+\frac{23}{37}+\frac{23}{38}+\frac{23}{39}$ eſt ad $-\frac{24}{34}-\frac{25}{35}+\frac{26}{36}+\frac{27}{37}+\frac{28}{38}+\frac{29}{39}$.
“ Eademque erit regula, continuatis tantum terminis, ſi non ſex ſed
“ decem vel plura puncta fixa ſupponerentur, qualia ſecundum me-

“ thodos Tangentium editas calculo præſtare, ſublatis irrationalibus,
“ tædioſiſſimæ & aliquando inſuperabilis operæ foret, ut ſi rectangula
“ plana vel ſolida ſecundum omnes biniones vel terniones poſſibiles
“ ex rectis illis compoſita datæ quantitati æquari deberent, in quibus
“ omnibus & multo implicatioribus, methodi noſtræ eadem eſt, opinione
“ multo major, rariſſimiq; exempli, facilitas; & hæc quidem initia ſunt
“ tantum Geometriæ cujuſdam multo ſublimioris ad difficillima &
“ puclherrima quæque etiam Miſtæ Matheſeos Problemata pertingentis,
“ quæ ſine Calculo noſtro differentiali aut * ſimili non temere quiſquam
“ pari facilitate tractabit. Appendicis loco placet adjicere ſolutionem
“ Problematis quod *Carteſius* a *Beaunio* ſibi propoſitum *Tom.* 3.
“ *Epiſt.* tentavit, ſed non ſolvit. Lineam invenire WW talis naturæ ut ducta
“ ad axem Tangente WC, ſit XC ſemper æqualis eidem rectæ conſtanti
“ $a$. Jam XW ſeu $w$ ad XC ſeu $a$, ut $dw$ ad $dx$, ergo ſi $dx$ (quæ aſſumi
“ poteſt pro arbitrio) aſſumatur conſtans ſeu ſemper eadem nempe
“ $b$, ſeu ſi ipſæ $x$ ſive AX creſcant uniformiter, fiet $W = \frac{a}{b}\,dW$ quæ erunt
“ ipſæ W ordinatæ, ipſis $dw$ ſuis incrementis ſive differentiis proportionales,
“ hoc eſt, ſi $x$ ſint Progreſſionis Arithmeticæ, erunt $w$ Progreſſionis
“ Geometricæ, ſeu ſi $w$ ſint numeri, $x$ erunt Logarithmi:
“ linea ergo WW Logarithmica eſt.

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* *Hic iterum veluti alius cujuſdam conſcius loquutus eſt.*

# CAP. VI.

## *De Methodo inveniendi Fluentes, &c. & ſpeciatim de Fluentibus Fluxionum Rationalium.*

CEleberrimus *Neutonus*, pag. ult. Tractatus ſui *De Quadraturis*, hæcce habet generalia de Fluentium (non ſolum rationalium ſed etiam ſenſu generaliſſimo quarumcunque) inveſtigatione, viz.

*Fluens pro Lubitu aſſumi poteſt, & aſſumptio corrigi ponendo Fluxionem Fluentis aſſumptæ æqualem Fluxioni propoſitæ, & terminos Homologos inter ſe comparando.* Anne vero egregia ſua Theoremata hacce methodo adinvenerit (quod facile crediderim) minime explicuerit, hiſce tamen verbis eadem inveniri poſſe ſatis plane innuerit; & hancce regulam quaſi *Criterion* veritatis concluſionis hoc modo elicitæ ibidem (pag. ſcil. præcedente) adfert. *Poſtquam Fluentes ex Fluxionibus collectæ ſunt, ſi de veritate concluſionis dubitatur, Fluxiones Fluentium inventarum viciſſim colligendæ ſunt, & cum Fluxionibus ſub initio propoſitis comparandæ; nam ſi prodeunt æquales, concluſio recte ſe habet: ſin minus, corrigendæ ſunt Fluentes ſic, ut earum Fluxiones Fluxionibus ſub initio propoſitis æquentur.* Et quoniam *Fluens omnis* (uti in ead. pag. penult. notat) *quæ ex Fluxione prima colligitur augeri poteſt vel minui quantitate quavis non Fluente* (quæ ſcil. colligendo Fluxiones methodo directa evanuerit) *quæ ex Fluxione ſecunda quantitate quavis cujus Fluxio tertia nulla eſt, & ſic in infinitum.* Jam ſi cui quantitatem iſtam reſtaurare ſit animus; ea conſulat quæ hac de re habent *Craigius* & *Cheynæus*, alter quidem pag. 38. Hic vero pag. 65. *Method. Inverſ.* & pag. 9 *Rudim.* ſcil. *Obtinetur ea quantitas ponendo Abſciſſam vel etiam ipſam Fluentem,* (per methodos infra tradendas) *repertam* = 0, *& habebis ſignum, iterumque ponendo indeterminatam quantitatem qua Fluens illa exprimitur* = 0, *obtinebis ipſam quantitatem adjiciendam Fluenti antea repertæ ſub contrario ſigno prout ſignum innuerit.* Hujuſce rei rationem vid. pag. 9. *Rudim. Cheynæi,* & exemplum prædict. pag. 65. *Meth. Inverſæ.*

Aſſumptiones autem, ſeu Fluentes aſſumendæ, pro Fluxionibus quibuſcunque ex *genio Fluxionis* ſeſe ſatis luculenter produnt, uti innuerit *Cheynæus,* & in ſimplicibus quidem ſeu vinculo non affectis Fluens obtinetur (ut verbis *Cheyneanis* utar) *operationes inſtituendo contrarias iis, quibus e Fluentibus eliciuntur Fluxiones.* Sic e. g. F : $mx^{m-1}\dot{x} = \frac{mx^{m-1+1}\dot{x}}{m+1-1\dot{x}} = x^m \pm a^m$, &c. & Fluens $my^{\pm n}x^{m-1}\dot{x} \pm nx^m y^{\pm n-1}\dot{y} = x^m y^{\pm n} \pm a^{m\pm n}$; hoc eſt, pro $mx^{m-1}\dot{x}$ Aſſumatur $Ax^m$ cujus accepta Fluxio æquetur Fluxioni datæ, ſcil. $mAx^{m-1}\dot{x} = mx^{m-1}\dot{x}$. Unde comparatione facta habebis $A = \frac{m}{m} = 1$. & Fluens erit $x^m \pm a^{m\pm n}$ &c. & eodem modo pro altera aſſumenda eſſet $Ax^m y^{\pm n}$ &c. & ſic ſi daretur $m - 1 \times m\dot{x}^2 x^{m-2} + m\ddot{x}x^{m-1}$ aſſumenda eſſet $A\dot{x}x^{m-1}$ unde methodo directa proveniret $m - 1A\dot{x}^2x^{m-2} + A\ddot{x}x^{m-1}$ & inveniretur $\overline{m-1}A + 1A = \overline{m-1\times m} + 1m$ & $A = \frac{\overline{m-1\times m}+1m}{m-1+1} = m$, & $A\dot{x}x^{m-1} = m\dot{x}x^{m-1}$ & Fluens abſoluta erit $x^m$, & ſic ad infinitum. Semper autem requiritur ut Fluentes iiſdem, quibus Fluxiones, indeterminatis (ſi id poſſibile ſit) exprimantur.

In Fluxionibus ſub vinculo ſubvolutis aſſumptio vel generalis vel particularis ex indice vel generali vel particulari plerumque determinatur; Et uti in æquationibus vulgaribus ſeu Algebraicis aſſumenda eſſet Æquatio generaliſſima ſeu infinita, & (omiſſis ſignis ex particulari ſupplendis ſeu determinandis) generaliter ſolvenda, vel abſolute, vel per ſeries infinitas, ut inde proveniret Canon generalis particularibus omnibus poſtea ſolvendis ſubſerviens, ad quamque particulares omnes facillime expenderentur; ita Fluens quævis in ſuo genere generaliſſima (e Fluxione generali proveniens) Fluxionum omnium particularium ſub ea forma generali contentarum Fluentes particulares (ordinatis ordinandis) exhibens, aſſumenda erit, ut ad eam expendantur, & determinentur, quod multum laboris & temporis, quinetiam & inanis ſæpiuſcule ſcrutaminis in Fluentibus particularibus indagandis, eaſdem ſtatim largiendo, compendioſe prævenerit. Hunc ad modum clariſſimus *Neutonus*, in excellentiſſimo ſuo opere, formas generales pro Rationalibus prius condidit, deinde particulares ad eaſdem expendit & abſolvit, *Quem* in hacce methodo ubique inſecutus eſt *Cheynæus*; quantoque facilior, atque adeo præſtantior, hujuſmodi expenſio particularis ad generalem, in Fluentibus inveſtigandis evaſura ſit, præ altera illa methodo particularem Fluentem independenter inveſtigandi, nemo erit in hiſce exercitatior, quin facile agnoverit; Aſſumptio quippe primaria & generaliſſima ſemper regularis eſt, ideoque facillima, & ſiquidem plures infinitatis gradus involvat, naturaliter Aſſumptiones ſubſequentes ſeſe e Primariæ Fluxione explicant ac largiuntur; & ex hac Fluente generali particulares omnes (ſub eo genere) facillime eruuntur, uti poſthac videbimus.

Sed jam ad *Neutonum* & Quadraturas redeamus; notandum autem interim magnum ſæpiuſcule diſcrimen eſſe inter Fluentem Fluxionis generalis Algebraicam, & Quadraturam curvæ, poſterior ſiquidem limitatior valde eſſe poteſt, & reſtrictior, uti etiam Tyronibus facile conſtabit. *Neutonus* in Tractatu prædicto (ut verbis *Walliſianis* utar) in curvis parallelis ordinatis gaudentibus *pro Fluxione abſciſſæ uniformi poſita*, unitatem aſſumit & exinde ordinatam ponit pro Fluxione Areæ; id eſt, poſita $x$ vel $z$ pro abſciſſa, & data convenienti expreſſione areæ $=v$, invenit (methodo directa) ejuſdem Fluxionem $=\dot{v}$, quæ exinde poſita $\dot{x}$ vel $\dot{z}=1$, dat ordinatam curvæ; & huc tendunt Propoſitiones Tertia & Quarta prædicti Tractatus, poſt præmiſſam ſecundam de inventione curvarum, *parallelis ordinatis gaudentium, quæ quadrari poſſunt.*

Sit *v.g.* in Prop. tertia $R=e+fz^{n}+gz^{2n}$ &c. Sitque $z^{\theta}R^{\lambda}=v$ $=$ areæ curvæ datæ; hoc eſt $z^{\theta}\times\overline{e+fz^{n}+gz^{n}\ \&c.}|^{\lambda}=v$. Capiantur utrinque Fluxiones & habebis $\theta\dot{z}z^{\theta-1}R^{\lambda}+\lambda z^{\theta}\dot{R}R^{\lambda-1}=\dot{v}$; hoc eſt, poſitis (more *Neutoniano*) jamjam memorato $RR^{\lambda-1}=R^{\lambda}$ in primo

Membro, $zz^{\theta-1} = z^{\theta}$ in ſecundo, habebis $\overline{\theta\dot{z}R + \lambda z\dot{R}}| \times z^{\theta-1} R^{\lambda-1} = \dot{v} =$ Fluxioni Areæ, quæ explicata, factaque $\dot{z} = 1$, dat $\theta e + \overline{\theta + \lambda n}\, fz^{n} + \overline{\theta + 2\lambda n}\, gz^{2n} + \overline{\theta + 3\lambda n}\, hz^{3n} +$ &c. $\times z^{\theta-1} R^{\lambda-1} =$ Ordinatæ. *Q. E. O.*

Sic in ſecundo caſu (ſeu *Prop.* quarta) poſita inſuper $S = k + lz^{n} + mz^{2n}$ &c. erit $z^{\theta} R^{\lambda} S^{\mu} = v$. Et capiendo Fluxiones, & ordinando æquationem ut ſupra, habebis $\overline{\theta\dot{z}RS + \lambda z\dot{R}S + \mu zR\dot{S}}| \times z^{\theta-1} R^{\lambda-1} S^{\mu-1} = \dot{v}$, quæ explicata & ordinata, factaque ubique $\dot{z} = 1$, dat Ordinatam quæſitam. Et eodem modo procedendum eſſet, ſi foret jam $T = p + qz^{n} + z^{2n} r$&c. & eſſet $z^{\theta} R^{\lambda} S^{\mu} T^{\nu} = v =$ Areæ Curvæ; & ſic porro ad infinitum: Hoc eſt, *Data Area quavis Curvæ ſeriebus quotcunque* (hoc modo) *expreſſa, Invenire, methodo directa, Fluxionem Areæ* (Fluxionem abſciſſæ unitati æqualem ponendo) *idem eſt ac invenire Ordinatam*; unde regrediendo, *Data, Curvæ cujuſcunque parallelis ordinatis gaudentis Ordinata, eandem in Fluxionem abſciſſæ ducere, ac proinde invenire Fluentem, idem eſt ac invenire aream, ſeu curvam quadrare.* Data autem Ordinata in Fluxionem abſciſſæ ducta, idem eſt quod apud D. *Gregorium* & alios *Elementum Curvæ* audit. Data autem tali Fluxione, aſſumenda erit (ſecundum regulam *Neutoni* ſuperius allatam) Fluens *idonea*, hoc eſt (verbis *Cheyneanis*) *Talis cujus Fluxio cum Fluxione data comparari poſſit*; comparatio autem (cujus leges jam a *Carteſio* ſatis notæ ſunt) inter terminos homologos procedit, ſeu (alterius cum altero) ejuſdem dimenſionis Algebraicæ; Aſſumenda, inquam, erit talis Fluens Coefficientibus indeterminatis adfecta, cujus reperta Fluxio cum Fluxionis datæ terminis comparari poſſit, ut inde determinentur Coefficientes aſſumpti, qui determinati Coefficientes loco indeterminatorum in Fluente aſſumpta ſubſtituti dabunt Fluentem quæſitam. Et huc tendunt ea quæ Propoſitione quinta & ſexta demonſtravit Vir celeberrimus. Sit e. g. in Prop. 5. ordinata curvæ $= z^{\theta-1} \times R^{\lambda-1} \times \overline{a + bz^{n} + cz^{2n}}$ &c. (exiſtente ſcil. uti prius $R = e + fz^{n}$

$+ gz^{2n}$ &c.) ſeu quod idem eſt ſic exprimatur $R^{\lambda-1} \times \overline{az^{\theta-1} + bz^{\theta+n-1}}$ $\overline{+ cz^{\theta+2n-1}}$ &c. erit jam $R^{\lambda-1} \times \overline{a\dot{z}z^{\theta-1} + b\dot{z}z^{\theta+n-1} + c\dot{z}z^{\theta+2n-1} + d\dot{z}z^{\theta+3n-1}}$ &c. = Fluxioni areæ, vel (uti loquitur *Cheynæus*) Fluxioni datæ, Ponatur autem ſeries $a\dot{z}z^{\theta-1} + b\dot{z}z^{\theta+n-1}$ &c. $= \dot{q}$, erit $\dot{q}R^{\lambda-1}$ = Fluxioni areæ. Aſſumatur jam $Az^{\theta} + Bz^{\theta+n} + Cz^{\theta+2n}$ &c. = Q = parti Fluentis aſſumptæ rationali indeterminatis Coefficientibus adfectæ; quæ ducta in $\overline{e + fz^n + gz^{2n}}|^{\lambda}$ dat Fluentem aſſumendam $= QR^{\lambda}$. Hujus Fluentis aſſumptæ (per methodum directam) capiatur Fluxio $\dot{Q}R^{\lambda}$ $\lambda Q\dot{R}R^{\lambda-1} = \dot{q}R^{\lambda-1}$, hoc eſt ad formam comparationi accommodam reducta $\overline{\dot{Q}R + \lambda Q\dot{R}}|\, R^{\lambda-1} = \dot{q}R^{\lambda-1}$ quæ exinde applicata ad partem irrationalem utrinque communem huncce dat Canonem $\dot{Q}R + \lambda Q\dot{R} = \dot{q}$. Qui explicitus & comparatus Coefficientes aſſumptas determinabit, quæ indeterminatarum loco ſubſtitutæ dabunt Fluentem quæſitam: e. g. in Propoſitione quinta *Newtoniana* $\overline{\dot{Q}R + \lambda Q\dot{R}}| \times R^{\lambda-1} = \dot{z}z^{\theta-1} \times \overline{a + bz^n + cz^{2n}}$ &c. $\times \overline{e + fz^n \text{ \&c.}}|^{\lambda-1}$; & dividendo utrinque per $R^{\lambda-1} = \overline{e + fz^n \text{ \&c.}}|^{\lambda-1}$ habebis $\dot{Q}R + \lambda Q\dot{R} = \dot{q} = a\dot{z}z^{\theta-1} + b\dot{z}z^{\theta+n-1} + c\dot{z}z^{\theta+2n-1}$ &c. hoc eſt terminis explicitis

$$\theta eA\dot{z}z^{\theta-1} + \left.\begin{matrix}\theta + n\,eB\\ \theta Af\\ \lambda n Af\end{matrix}\right\} \dot{z}z^{\theta+n-1} \text{ \&c.} = a\dot{z}z^{\theta-1} + b\dot{z}z^{\theta+n-1} \text{ \&c.}$$

Hinc autem 1°. $\theta eA = a$; Unde $A = \frac{a}{\theta e}$.

2°. $\theta + n,\ eB + \theta Af + \lambda n Af = b$ ſeu $\overline{\theta + n},\ eB = b - \theta Af - \lambda n Af$.

Unde $B = \frac{b - \overline{\theta + \lambda n}, fA}{\overline{\theta + n}, e}$ pone jam uti *Neutonus* $\frac{b}{n} = r$, &c. & fac ſubſtitutiones debitas in æquatione aſſumpta, & habebis aream quæſitam terminis *Neutonianis* expreſſam. *Q. E. I.*

Huc recidit caſus Infiniti-nomialis pag. 12. *Cheynæi* (utpote qui idem eſt cum hocce *Neutoniano)* viz. poſitis $z^{r-1} \dot{z} \times \overline{a + bz^{n} + cz^{2n}}$ &c. $= \dot{q}$, & $\overline{e + fz^{n} \text{ \&c.}}|^{m-1} = R^{\lambda-1}$. Aſſumatur ut ſupra (poſita prius $\frac{1}{p} = m$) $QR^{\frac{1}{p}}$; erit $p\dot{Q}R + Q\dot{R} = \dot{q} = paz^{r-1} + pbz^{r+n-1}$ &c. (facta ſcil. $\dot{z} = 1$) ad infinitum. Et Fluxio Binomialis, e. g. hujuſce ſcil. caſus particularis, ubi $\overline{e + fz^{n}}|^{\lambda+1} = R^{\frac{1}{p}}$ dat pro comparatione $p\dot{Q}R + Q\dot{R} = pdz^{\theta}$. Et eodem modo pro Trinomiis, Quadrinomiis, &c. procedendum eſt ad infinitum (ſeu ad Infinitinomium, quod eſt in hocce genere ultimum) poſitis ſcil. Coefficientibus omnibus ultra particularis caſus dati ultimum æqualibus nihilo in Theoremate Infinitinomio; Inferiores enim quicunque, unuſquiſque ſcil. reſpective in ſuperiore quovis ſui generis, continentur.

Sit jam uti in Propoſitione *Neutoniana* ſexta, Fluxio data $\dot{q}R^{\lambda-1}S^{\mu-1}$ ubi $s = k + lz^{n} + mz^{2n}$ &c. Aſſumatur $QR^{\lambda} S^{\mu} =$ Fluenti; cujus accepta Fluxio $\overline{\dot{Q}RS + \lambda Q\dot{R}S + \mu QR\dot{S}}| R^{\lambda-1} S^{\acute{\alpha}-1} = \dot{q} R^{\lambda-1} S^{\mu-1}$ ſeu $\dot{Q}RS + \lambda Q\dot{R}S + \mu QR\dot{S} = \dot{q}$ pro comparatione inſtituenda; & terminis explicatis ac comparatis, ſubſtitutiſque ſubſtituendis, prodibit Fluens quæſita. Eodem prorſus modo generaliter ad infinitum, ſi poneretur $T = p + qz^{n} + rz^{2n} + sz^{3n}$ &c. & foret Fluxio data

$\dot{q}R^{\lambda-1}S^{\mu-1}T^{\nu-1}$ aſſumenda eſſet $QR^{\lambda}S^{\mu}T^{\nu}$ pro Fluente, & ſic ad infinitum. Et hinc videre licet quod olim inſinuaverit *Neutonus* ad *Walliſium*, *Se* ſcil. *nihil in Quadraturis generale invenire potuiſſe antequam rem ad nudam conſiderationem Ordinatarum reduxiſſet.* Harum omnium Quadraturarum demonſtrationem *Neutonus* contexuit ponendo Ordinatarum particularium (ſua methodo Erutarum) collectam ſummam æqualem Ordinatæ generali Curvæ ex omnibus conflandæ, & æquando terminos correſpondentes (h. e. comparando terminos homologos) ut determinentur Coefficientes aſſumptæ, & facta ſubſtitutione debita, prodibit Area quævis propoſita. Pro inferiorum ordinum Fluxionibus quæ (numero infinitæ) ſub generalium harum formarum unaquaque continentur, ſive *Bivinculares* ſive *Trivinculares*, &c. ad infinitum exiſtant; ſive etiam ſub quavis harum *Binomiæ* vel *Trinomiæ*, &c. ad infinitum, Aſſumenda erit æquatio ex *Nomio* unoquoque unitate aucto, & in ſeriem Rationalem ducta Coefficientibus indeterminatis adfectam, & procedendum ut ſupra pro Fluente. Exempla videre eſt in *Cheyn.* pag. 9. pro *Binomiali Neutoniana*, pag. 68, pro *Trinomiali*, &c. ubi etiam regulæ occurrunt pro aſſumptione Indicum. Sed ad *Neutonum*, h. e. ad ipſum fontem, revertamus; Circa praxin hæc habet.

† *Omnis* (inquit) *Ordinata* (pag. 179.) *poteſt duobus modis in ſeriem reſolvi.*

1. *Ubi Index n Affirmativus eſt.*
2. *Ubi Negativus eſt.*

*Sit v. g. Ordinata* $\frac{3k - lzz}{zz\sqrt{kz - lz^3 + mz^4}}$ *hæc vel ſic ſcribi poteſt, viz.*

$z^{-\frac{5}{2}} \times \overline{3k - lzz} \times \overline{k - lz^2 + mz^3}|^{\frac{1}{2}}$; *vel ſic* $z \times -l + 3kz^{-2} \times \overline{m - lz^{-1} + kz^{-3}}|^{-\frac{1}{2}}$.

Quod etiam *Tyro* ſic facile colliget. Primo, ſi ponatur

$\frac{3k - lz^2}{z^2\sqrt{kz - lz^3 + mz^4}} = y$, erit $3k - lz^2 = yz^2\sqrt{kz - lz^3 + mz^4}$, &

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† *Vide etiam* Cheyn. *p.* 66.

$3k - lz^2 = yz^{\frac{4}{2}} \times z^{\frac{1}{2}} \sqrt{k - lz^2 + mz^3} = yz^{\frac{5}{2}} \times \sqrt{k - lz^2 + mz^3}$ hoc est $z^{-\frac{5}{2}} \times \overline{3k - lz^2} \times \overline{k - lz^2 + mz^3}|^{\frac{1}{2}} = y$. Quod eſt primum. Vel ſic etiam $\frac{3k - lzz}{zz\sqrt{kz - lz^3 + mz^4}} = z^2\sqrt{z}\sqrt{k - lzz + mz^3} = \sqrt{z^4}\sqrt{z}\sqrt{k - lzz + mz^3}$ $= \frac{3k - lz^2}{\sqrt{z^5}\sqrt{k - lz^2 + mz^3}}$, hoc eſt $\frac{3k - lzz}{\sqrt{z^5}\sqrt{k - lz^2 + mz^3}} = y = z^{-\frac{5}{2}} \times \overline{3k - lzz}$ $\times \overline{k - lzz + mz^3}|^{-\frac{1}{2}}$. Secundo, $z^{-\frac{5}{2}} \times \overline{3k - z^2} \times \overline{k - lz^2 + mz^3}|^{-\frac{1}{2}}$ $= z \times \overline{3kz^{-2} - l} \times \overline{kz^{-3} - lz^{-1} + m}|^{-\frac{1}{2}}$. Quod ſic eluceſcet,

1°. $z^{-\frac{5}{2}} \times \overline{3k - lz^2} = \frac{3k - lzz}{\sqrt{z^5}} = \frac{3k - lzz}{zz\sqrt{z}} = \frac{3kz^{-2} - l}{\sqrt{z}}$.

2°. $\overline{kz - lz^2 + mz^3}|^{-\frac{1}{2}} = \sqrt{z^3} \times \overline{kz^{-3}\ lz^{-1} + m}|^{-\frac{1}{2}} = \frac{\sqrt{z^3}}{\overline{kz^{-3} - lz^{-1} + m}|^{\frac{1}{2}}}$.

3°. $\frac{3kz^{-2} - l}{\sqrt{z}} \times \frac{\sqrt{z^3}}{\overline{kz^{-3} - lz^{-1} + m}|^{\frac{1}{2}}} = \frac{\sqrt{z^3} \times \overline{3kz^{-2} - l}}{\sqrt{z} \times \overline{kz^{-3} - lz^{-1} + m}|^{\frac{1}{2}}} =$

4°. $\frac{\sqrt{z^3}}{\sqrt{z}} \times \frac{\overline{3kz^{-2} - l}}{\overline{kz^{-3} - lz^{-1} + m}|^{\frac{1}{2}}} = \frac{\sqrt{z^2}}{1} \times \frac{3kz^{-2} - l}{\overline{kz^{-3} - lz^{-1} + m}|^{\frac{1}{2}}} =$

5°. $z \times \overline{3kz^{-2} - l} \times \overline{kz^{-3} - lz^{-1} + m}|^{-\frac{1}{2}}$. Quod eſt ſecundum.

Et hiſce vel ſimilibus modis reducentur Ordinatæ ad formam Canoni accommodam. *Tentandus* ( inquit *Neutonus* ) *eſt caſus uterque, & ſi ſerierum alterutra ob terminos tandem deficientes abrumpitur ac terminatur, habebitur Area Curvæ in terminis finitis.* Sit jam Ordinata prædicta & reducta $z^{-\frac{5}{2}} \times \overline{3k - lz^2} \times \overline{k - lz^2 + mz^3}|^{\frac{1}{2}}$ quæ æquetur ordinatæ generali in Canone, viz. $z^{-\frac{5}{2}} \times \overline{3k - lz^2} \times \overline{k - lz^2 + mz^3}|^{\frac{1}{2}} = z^{\theta - 1}$

$$\times\,\overline{a+bz^{n}+cz^{2n}}\times\overline{e+fz^{n}+gz^{2n}+bz^{3n}\ \&c.}\big|^{\lambda-1}$$ ; hinc erit

$$\left.\begin{array}{l}\theta-1=-\frac{5}{2}.\ \theta=-\frac{3}{2}=r.\\ \lambda=-\frac{1}{2}.\ \lambda-1=\frac{1}{2}.\ n-1.\\ s=-1.\ t=-\frac{1}{2}.\ v=-0.\end{array}\right\}\begin{array}{ll}3k=a. & k=e.\\ 0=b. & 0=f.\\ -l=g. & b=m\\ & \&c.\end{array}$$

& Area $z^{\theta}R^{\lambda}\times\dfrac{\frac{1}{n}a}{re}$. Unde

$$R=k-lz^{2}+mz^{3}=e+(\,0=f\,)\,z^{n}+gz^{2n}+bz^{3n}.$$

$$\&\ z^{\theta}R^{\lambda}=z^{-\frac{3}{2}}\times\overline{k-lz^{2}+mz^{3}}\big|^{\frac{1}{2}}.$$

$$\&\ \frac{\frac{1}{n}a}{re}=\frac{\frac{1}{1}3k}{-\frac{3}{2}k}=-\frac{3}{2}\Big)\frac{3}{1}\Big(-\frac{6}{3}=-2.$$

$$\text{Et Area }z^{\theta}R^{\lambda}\times\frac{\frac{1}{n}a}{re}=-2\sqrt{k-lz^{2}+mz^{3}}.$$ *Q. E. I.*

* *Et hæc Area adjacet Abſciſſæ ultra Ordinatam productæ* : Hoc eſt, ad inferiores Curvæ partes. *Nam Area omnis affirmativa adjacet tam Abſciſſæ quam Ordinatæ, negativa vero cadit ad contrarias partes Ordinatæ & adjacet Abſciſſæ productæ, manente ſcil. Signo Ordinatæ.*

*Et hoc modo Series alterutra ſemper,* & *nonnunquam utraque terminatur, & finita evadit ſi Curva Geometrice quadrari poteſt. At ſi Curva talem Quadraturam non admittit, Series utraq; continuabitur in inf. & earum altera converget, & Aream dabit approximando, præterquam,* 1°. Ubi $\frac{\theta}{n}=r$ (propter Aream infinitam) vel nihil eſt, vel numerus integer & negativus. Vel 2° Ubi $\frac{z^{n}}{e}=1$. h. e. ubi $z^{n}=e$ ; quæ conditiones ſunt non-convergentiæ ; conditiones autem convergentiæ ſunt

1°. Si $\frac{z^{n}}{e}$ unitate minor eſt, & tum converget ſeries in qua Index $n$ affirmativus eſt : & in hoc caſu Area adjacet Abſciſſæ ad uſque Ordinatam ductæ. Sin

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* *Pag.* Neut. *Edit. Anglic.* 179. *Pag.* Cheyn. 66.

2°. Si $\frac{z^n}{e}$ unitate major eſt, converget ſeries altera; & tunc Area adjacet Abſciſſæ ultra Ordinatam productæ.

Sequuntur in *Neutono* notæ quædam pro Ordinatis, quæ hujuſmodi ſunt.

Sit in Ordinata quacunque $P$ = Numeratori.
$Q = \left\{ \right.$ Vel Factori cuivis, vel Denominatori, rationali.
$R^\pi$ = Factori ſurdo irreducibili.
$R$ = Ejuſdem lateri.
$S^\mu$ & $S$ ut & $T^\nu$ & $T$ = Factoribus ſurdis & later. eorundem ſecundo & tertio.

## *HINC FORMÆ ORDINATARUM.*

| *Prima ad inf. pergens.* | *Secunda ad inf. pergens.* |
|---|---|
| 1. $QR^\pi$ | 1. $\frac{P}{Q}$ |
| 2. $QR^\pi S^\mu$ | 2. $\frac{P}{QR^\pi}$ |
| 3. $QR^\pi S^\mu T^\nu$ &c. ad inf. | 3. $\frac{P}{QR^\pi S^\mu}$ |
| | 4. $\frac{P}{QR^\pi S^\mu T^\nu}$ &c. ad inf. |

*Caſ.* 1. *Si Ordinata contentum eſt ſub* $QR^\pi$, *&* R *non dividit* Q, erit $\lambda - 1 = \pi$. *&* $R^{\lambda-1} = R^\pi$.

*Sin* R *dividit* Q *ſemel, erit* $\lambda - 1 = \pi + 1$ *&* $R^{\lambda-1} = R^{\pi+1}$.

*bis*, . . . . $\lambda - 2 = \pi + 2$ *&* $R^{\lambda-1} = R^{\pi+2}$.

*ter*, . . . . $\lambda - 3 = \pi + 3$ *&* $R^{\lambda-1} = R^{\pi+3}$.

& ſic deinceps.

*Caſ.* 2. *Si Ordinata ſit fractio rationalis irreducibilis cum denominatore e duobus vel pluribus terminis compoſito : Reſolvendus eſt denominator in ſuos omnes diviſores Primos, Et ſi diviſor ſit aliquis cui nullus alius eſt æqualis, Curva quadrari nequit : Sin duo vel plures ſint diviſores æquales, rejiciendus eſt eorum unus, & ſi adhuc aliis duo vel plures ſint ſibi mutuo æquales & prioribus inæquales, rejiciendus eſt etiam eorum unus, & ſic in aliis omnibus æqualibus ſi adhuc plures ſint : Deinde diviſor qui relinquitur, vel contentum ſub diviſoribus omnibus qui relinquuntur, ſi plures ſunt, ponendum eſt pro* R, *& ejus quadrati reciprocum* $R^{-2}$ *pro* $R^{\lambda-1}$ *præterquam ubi contentum illud eſt Quadratum vel Cubus, vel Quadrato-Quadratum, &c. Quo in Caſu ejus latus ponendum eſt pro* R, *& poteſtatis Index* 2 *vel* 3 *vel* 4, *negative ſumptus, pro* $\lambda$. *Et Ordinata ad denominatorem* $R^2$ *vel* $R^3$ *vel* $R^4$ *vel* $R^5$, *&c. reducenda.*

Ut ſi Ordinata ſit $\frac{z^5 + z^4 - 8z^3}{z^5 + z^4 - 5z^3 - z^2 + 8z - 4} = \frac{P}{Q}$.

*Quoniam hæc Fractio irreducibilis eſt & denominatoris diviſores ſunt pares, nempe* $z - 1$, $z - 1$, $z - 1$, *&* $z + 2$, $z + 2$ rejicit *magnitudinis utriuſque diviſorem unum & reliquorum contentum* $z - 1$, $z - 1$, $z + 2 = z^3 - 3z + 2$ ponit pro R unde erit

$$\frac{1}{z^6 - 6z^4 + 4z^3 + 9z^2 - 12z + 4} = \frac{1}{R^2} = R^{-2} \text{ ponendum pro } R^{\lambda-1}.$$

Dein Ordinatam ad denominatorem $R^2$ ſeu $R^{1-\lambda}$ reducit ſcil.

$$Q : P :: z^6 - 6z^4 + 4z^3 + 9z^2 - 12z + 4 \, (= R^2) : z^6 - 9z^4 + 8z^3 \left(= \frac{PR^2}{Q}.\right)$$

tunc $\frac{z^6 - 9z^4 + 8z^3}{Q} = z^3 \times \overline{8 - 9z + z^3} \times \overline{2 - 3z + z^3}|^{-2}$

Unde $a = 8$, &c.

Unde $z^\theta R^\lambda \times \frac{\frac{1}{n}a}{re} = z^4 R^{-1} \times \frac{\frac{1}{1}8}{4 \times 2} = \frac{z^4}{R} = \frac{z^4}{z^3 - 3z + z} =$ Areæ.

Terminis omnibus in tota Serie poſt primum evaneſcentibus.

Si denique Ordinata eſt Fractio irreducibilis, & ejus denominator contentum eſt ſub factore rationali Q & factore ſurdo irreducibili $R^{\pi}$, inveniendi ſunt lateris R diviſores omnes primi, & rejiciendus eſt diviſor unus magnitudinis cujuſque & per diviſores qui reſtant ſi qui ſint, multiplicandus eſt factor rationalis Q: Et ſi factum æquale eſt lateri R vel lateris illius poteſtati alicui cujus Index eſt numerus integer, eſto ille $m$, erit $\lambda - 1 = -\pi - m$, & $R^{\lambda-1} = R^{-\pi-m}$. Ut ſi Ordinata ſit

$$\frac{3q^5 - q^4x + 9q^3x^2 - q^2x^3 - 6qx^4}{qq - x^2\sqrt[3]{q^3 + q^2x - qx^2 - x^3}} = \frac{P}{QR^{\pi}}$$

quoniam jam factoris ſurdi latus $R = q^3 + q^2x - qx^2 - x^3$ diviſores habet $q + x$, $q + x$, $q - x$ qui duarum ſunt magnitudinum, rejicit diviſorem unum magnitudinis utriuſque & per diviſorem reliquum $q + x$ multiplicat factorem rationalem $q^2 - x^2 = Q$; et quoniam factum $q^3 + q^2x - qx^2 - x^3 = R$, ponit $m = 1$. Et inde cum $\pi$ ſit $\frac{1}{3}$, fit $\lambda - 1 = -\frac{4}{3}$. Ordinatam igitur reducit ad denominatorem $R^{-\frac{4}{3}}$ & fit $z^0 \times 3qx^6 + 2q^5x + 8q^4x^2 + 8q^3x^3 - 7q^2x^4 - 6x^5 \times \overline{q^3 + q^2x - qx^2 - x^3}|^{-\frac{4}{3}}$.

Unde (æquatis æquandis) ut deſumantur definitiones coefficientium Ordinatæ reductæ ad formam debitam a coefficientibus in prima Ordinata expoſita. Erit $3q^6 + 2q^5x + 8q^4x^2$, &c. $= a + bz^n + cz^{2n}$ &c. Et $\overline{q^3 + q^2x - qx^2 - x^3}|^{-\frac{4}{3}} = \overline{e + fz^n + gz^{2n}, \&c.}|^{\lambda - 1}$.
Unde $a = 3q^6$. $b = 2q^5$, &c. $e = q^3$. $f = qq$, &c.
$\theta - 1 = 0$. $\theta = 1 = n$. $\lambda = -\frac{1}{3}$. $r = 1$. $s = \frac{2}{3}$. $t = \frac{1}{3}$. $v = 0$. &c. &c.
Et Area prodit (ſubſtitutis ſubſtituendis) $\frac{3qqx + 2x^3}{\sqrt{a^3 + a^2x - ax^2 - x^3}}$ terminis omnibus in Serie tota poſt tertium evaneſcentibus.

Alteram inſuper Propoſitionem altioris in hiſce generis addit *Newtonus*, & Seriem earundem ad infinitum pergere ibidem aſſerit uti prius innuimus. Et annis aliquot poſt hæcce cum amicis quibuſdam per Epiſt. communicata uti *E. Wallis*, pag. 391, Edit. noviſſ. videre eſt D. *Gregorius* M. D. *Matheſeos Profeſſor tum temporis* Edinburg. *poſtea* Oxon. *Invenit per aliam Methodum Seriem ejuſdem generis cum Binomiali* Newtoniana *quanquam terminis complexionibus involutam*; *eamque D.* Archibaldus Pitcarnius *diebus tribus circiter vel quatuor antequam* Newtonianam *viderat, in lucem edidit.* & tria vel quatuor Exempla ſujungit quæ pro lubitu conſulat Lector, & ad hujuſmodi Series ad plurinomias etiam ſeu Infinitinomias quantitates extendi poſſe ibidem aſſerit.

# CAP. VII.

## *De Fluentibus Fluxionum Logarithmicalium, &c. Et Exponentialium.*

INventio *Exponentialium Fluxionum* uti & earundem cognatio cum Curvis Logarithmicis duplicem Authorem agnoſcit ſcil. *Leibnitium* (uti in Epiſtolis illis *Neutonianis* ad *Walliſium* videre eſt) et *Johannem Bernoullium.* Anno 1697 Menſe *Martii* (oblata occaſione a Domino *Nieuwentiit*) Geometra Clariſſimus *Johannes Bernoulli* in *Actis Eruditor.* de hujuſce generis Calculi Inventione & Principiis hæc habet.

*Hancce Inventionem eatenus* ſibi *attribuendam* dicit, *quod jam tum ante quinquennium ſponte ſua eo pervenerat, Domino* Leibnitio *non monente,* ut iſte (*Leibnitius* ſcil.) fatetur in actis Anni 1695 pag. 314.

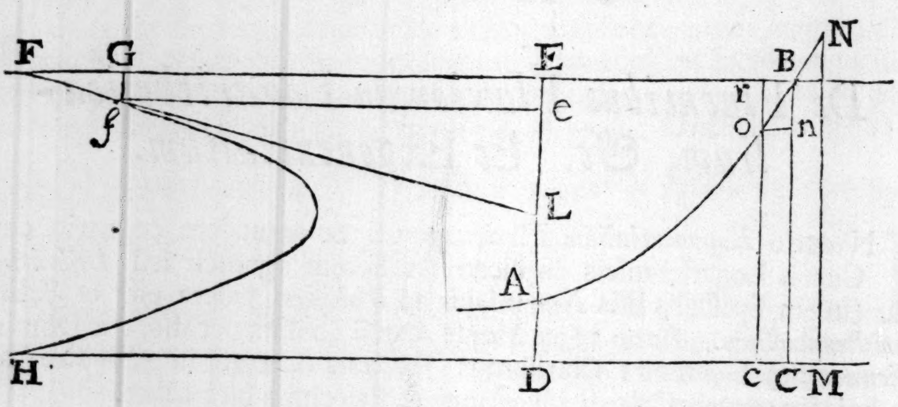
Ipſum interim *Leibnitium* eundem inveniſſe calculum & ipſe faſſus eſt; *Quantitatem,* inquit, *elevatam ad potentiam indeterminatam (antequam etiam hæc* Leibnitio *conſiderata Intellexiſſem) vocabam percurrentem, quia omnes poſſibiles dimenſiones quaſi percurrit, hinc æquationes quantitatibus hujuſmodi conſtantes, ipſæque Curvæ per illas deſignatæ, eodem mihi nomine veniebant, quoniam vero exponentialium appellatio magno noſtro Geometræ arriſit, in illius honorem hoc alterum nomen & ego aſciſcam, miſſo priore, ne quod ſubinde accidit diverſa ejuſdem rei denominatio Lectori confuſionem pariat. Exponentialem igitur quantitatem concipiebam utut medium quid inter Algebraicam & Tranſcendentem, accedit enim, ad Algebraicam eo quod terminis finitis utut Indeterminatis conſtat, ad Tranſcendentem vero quod nulla conſtructione Algebraica exhiberi poſſit. Ipſa Logarithmica (communium Tranſcendentalium ſimpliciſſima) eſt ex hoc Curvarum genere, omneſque aliæ exponentiales ope hujus conſtrui, & earum tangentes determinari poſſunt, ut ex jam dicendis patebit.* Deinde (in *Nieuwentytii* prædicti gratiam uti ait) hæc tria (licet alias ſatis trita) exponit ut ſequitur.

1. *Quomodo una eademque Indeterminata poſſit eſſe in Curvæ Logarithmica de applicatarum genere & ſimul abſciſſa alterius cujuſdam Curvæ.*

2. *Quomodo ex æquatione* $x^v = y$ *deducatur hæc æquatio* $vlx = ly$, ubi per $lx$, $ly$, $lz$, &c. Intelligit Logarithmos ipſarum $x$, $y$, $z$, &c.

3. *Quomodo lx poſſit eſſe integrale* (ſeu Fluens) $\frac{\dot{x}}{x}$.

1. *Eſto* DE *Axis Curvæ* HGI *illiq;* *perpendicularis* DC.



“ Axis Logarithmicæ AB; producatur Applicata FE donec occurret “ Logarithmicæ AB in B a quo demiſſa perpendicularis BC æqualis “ erit Abſciſſæ DE; eſt autem BC applicata Logarithmicæ; ergo “ patet qua ratione abſciſſæ DE, cujuſcunque Curvæ HGF poſſint eſſe “ de applicatarum genere non ſolum in Logarithmica ſed in quavis “ etiam alia Curva AB, & quidem ita ut ſalva maneat relatio inter “ DE & EF quocunque etiam modo Creſcat DE ſive æqualiter ſive inæ- “ qualiter, adeo ut Sumptis differentiis Axis Logarithmicæ Cc æqualibus “ nihilominus, Differentialis FG & Differentiales Ec ſeu ipſa FE ad “ Subtangentem EL ſuam generalem perpetuo ſervat relationem.

“ 2. Ad demonſtrandum ſecundum quod ſit $vlx = ly$ inſpiciendæ “ ſunt (inquit) Vulgares quæ proſtant Tabulæ Logarithmicæ, quæ “ docent Logarithmum numeri quadrati, Cubi, Biquadrati, &c. Eſſe “ duplum, triplum, quadruplum, Logarithmi radicis, unde generaliter “ Log. $x^v = vlx$, &c. Supereſt

“ Tertium; Quod ita Oſtendo; ſi AD $= 1 =$ Subtangenti, & BC $=$ “ numero, erit, ut conſtat, DC Logarithmus ejuſdem, eſt autem ex “ Natura Logarithmicæ Subtang. in B$n$ $=$ BC $\times$ C$c$, & per Conſequens

“ quens $\frac{Bn}{BC}$ = C*c*, hinc ſi BC ſit $x$, $y$, $z$, &c. Et DC $lx$, $ly$, $lz$, &c.
“ erit $\frac{\dot{x}}{x}$, $\frac{\dot{y}}{y}$, $\frac{\dot{z}}{z}$, &c. $= \varphi : lx$, $ly$, $lz$, &c. Ergo etiam
F: $\frac{\dot{x}}{x}$, $\frac{\dot{y}}{y}$, $\frac{\dot{z}}{z} = lx$, $ly$, $lz$, &c. unde &c. et Demonſtratio generalis “ eſt, & nulla conditione, nullo reſpectu limitata. Demonſtratum “ enim eſt quacunque ratione Creſcant DC, DE, ſeu *bc*, BC ; quocunque “ demum appellentur nomine ſive $x$ ſive $y$, ſive abſolute, ſive “ reſpective conſiderentur, perpetuo earum Logarithmos eſſe D*c*, DC. “ Quod autem aſſumatur AD = 1 = Subtangenti id Compendii gratia “ tantum fit, quamcunque enim quantitatem, quidni ergo & Subtangentem “ unitati ſubſtituere tuto licet, ſi modo ad aſſumptum cæteræ “ omnes referantur. Diſcuſſis jam ut opinor difficultatum Nebulis “ venio ad ipſam Exponentialium tractationem. Quantitates Exponentiales “ ſunt diverſorum graduum quorum Infimus (qui hactenus tantum “ fuerat conſideratus) eſt quando Exponens conſtat Indeterminatis “ Ordinariis ut $y^m$, $x^n$, $z^p$ poſſint $^{m,\ n,\ p}$ eſſe quantitates ſimpliciter Indeterminatæ.

“ Quantitas Exponentialis ſecundi gradus eſt, cum Exponens ipſe “ eſt quantitas Exponentialis ut $y^{m^n}$ ; & ſic in Univerſum quantitas “ Exponentialis cujuſcunque gradus habet pro Exponente quantitatem “ Exponentialem gradus proximè præcedentis. Idem etiam Intellige “ de æquationibus & Curvis Exponentialibus ad ſuos gradus referendis. “ Nota interim quod ſi æquatio conſtet ex diverſis diverſorum graduum “ Exponentialibus, tunc æquatio ut & Curva per illam deſignata “ ſortietur nomen a potiori, de quibus omnibus hæc habe.

“ Eſto Curva Logarithmica quævis AB ſumptâque Subtangente pro “ Unitate, & ipſi æquali applicata AD ; Ordinata B*b* quovis modo “ varians, ſive ſit de Applicatarum Serie, ſive de Abſciſſarum genere “ alterius cujuſcunque Curvæ, ſi modo per minima Creſcat, ſive æqualiter “ ſive inæqualiter, ſit $x$, $y$, $z$, adeoque DC $= lx$, $ly$, $lz$. B*n* = “ $\dot{x}$, $\dot{y}$, $\dot{z}$. C*c* $= \varphi : lx$, $ly$, $lz$, &c. Unde fluit regula generalis *Fluxio* “ *logarithmi utcunque Compoſiti eſt æqualis fluxioni numeri diviſæ per Nu-*

*merum, ut* $\varphi : l : \sqrt{xx+yy} = \frac{x\dot{x}+y\dot{y}}{xx+yy}$. Ad Inveniendam ergo fluxionem
“ quantitatis Exponentialis primi gradus $m^n$ fiat $m^n = t$. Ergo $nlm = lt$;
“ at per Regulam generalem $\varphi : lm = \frac{\dot{m}}{m}$ & $\varphi : lt = \frac{\dot{t}}{t}$ ideoque $\frac{n\dot{m}}{m}$
$+ lm\dot{n} = \frac{\dot{t}}{t} =$ (ob. $m^n = t$) $\frac{\dot{t}}{m^n}$; unde $\dot{t}$ ſeu $\varphi : m^n = nm^{n-1}\dot{m} + m^n lm\dot{n}$;
“ id quod ſuggerit Regulam primam ſpecialem pro Exponentialibus
“ primi gradus, poſſunt enim pro $m$ & $n$ Quantitates intelligi quomo-
“ docunque ex Indeterminatis compoſitæ. Eſto jam Exponentialis
“ ſecundi gradus $m^{n^p}$ ponatur illa æqualis $t$, adeoque $n^p lm\dot{n} = lt$, ſump-
“ tis utrobique fluxionibus modo communi $n^p \varphi : l : m + l : m \varphi : n^p =$
“ $\varphi : l : t$. Quoniam autem per Regulam primam ſpecialem $\varphi : l,m$
“ $= \frac{\dot{m}}{m}$ & $\varphi : l,t = \frac{\dot{t}}{t}$ habebitur Regula ſecunda pro Exponentialibus
“ ſecundi gradus quæ hæc eſt, $\varphi : m^{n^p} = n^p m^{n^p-1}\dot{m} + pn^{p-1}m^{n^p}lm\dot{n}$
“ $+ n^p m^{n^p} lmln\dot{p}$. Eodem modo Inveniuntur Regulæ ſequentes pro alio-
“ rum graduum Exponentialibus, nec difficilius eſt Invenire Fluxiones
“ quantitatum quomodocunque ex illis compoſitarum ut $\varphi : m^n p^q = p^q$
“ $\times \varphi : m^n + m^n \times \varphi : p^q$ ubi ſi ſurrogetur valor ipſarum $\varphi : p^q$, $\varphi : m^n$ modo
“ ſupra Inventus prodibit Fluxio quæſita.

## *Exempla quæ adfert Vir Laudatus ſunt hæc.*

“ 1. Conſtruenda eſt Curva cujus Natura Exprimitur per hanc
“ æquationem Exponentialem $x^x = y$; ejus Tangens determinanda
“ Sumptis Logarithmis habetur $xlx = ly$ facta itaque Logarithmicâ AB
“ cujus ſubtangens ſit $= 1 =$ AD ſitque DE vel BC $= x$, erit DC $= lx$;

&

“ & per Conſequens ſi fiat AD : BC :: DC : DM erit DM $= ly$ & “ MN $= y$, cui ſi æqualis applicetur EF ad Abſciſſem DE erit punctum F in Curva optata HGF, Tangens in F determinatur ſic per “ Regulam primam ſpecialem $\dot{y} = x^x\dot{x} + x^x l : x\dot{x} =$ (ſubſtituto $y$ loco “ $x^x$) $y\dot{x} + 9lx\dot{x}$ id eſt $y + ylx : 1 :: \dot{y} : \dot{y}$ : ſubtang. $\frac{1}{1 + lx}$ hinc ſumpta “ EL tertia proportionali ad AD + DC & AD, erit FL tangens; “ quam proinde in terminis ordinariis expreſſimus.

“ Per Ordinarium & Aſſignabile non ſolum Algebraicam ſed etiam “ Tranſcendentem quantitatem, ſi modo ſit finita, comprehendit *Leibnitius*; Circuli utique peripheria terminis algebraicis Exprimi nequit, “ an ideo non eſt Ordinaria ſeu finita? ad ideo minus aſſignabilis, cum “ tamen Unico ductu Circini aſſignetur.

“ 2. Si proponatur Curva cujus æquatio $x^y = a$; habebitur ad Logarithmos reducta $ylx = la$, unde ut ſupra per Logarithmicam factum “ eſt, Elicitur Curvæ conſtructio; fluxiones autem inveniendo per “ Reg. 1. Specialem obtinetur $\dot{x} + lx\dot{x} = ly\dot{y}$ ex quo pariter Subtang. “ determinatio fluit, Obſervatu dignum hic venit quod hujus Figuræ “ Quadratura citra Seriem poſſit exhiberi, eſt enim $F : y\dot{x} = \frac{2xxlx - xx}{la}$.

“ 3. Si proponeretur $a^x = y$; adeoque $xla = ly$ prodiret $yla\dot{x} = \dot{y}$; “ id eſt Subtangens foret $= \frac{1}{la} =$ conſtanti; unde concludendum “ æquationem propoſitam ipſiſſimæ Logarithmicæ competere. Unico “ ergo ductu calami planam Regiamque qualem *D, N.* deſiderat viam “ aperui Logarithmicam hoc tempore nulli non cognitam ad æquationem Exponentialem reducendi.

“ 4. Eſto jam Exponentialis plurium terminorum, e. g. $x^x + x^a$ “ $= x^y + y$; ſumptis ſeparatim fluxionibus cujuſque termini per Regulam primam ſpecialem reperietur $\varphi : x^x = x^x\dot{x} + x^x lx\dot{x}$. $\varphi : x^a =$ “ $ax^{a-1}\dot{x}$. $\varphi : x^y = yx^{y-1}\dot{x} + x^y lx\dot{y}$ quæ rite diſpoſita dabunt æquationem hanc $x^x\dot{x} + x^x x\dot{x} + ay^{a-1}\dot{x} - yx^{y-1}\dot{x} = x^y lx\dot{y} + \dot{y}$; facto itaq; “ uti $\dot{y}$ ad $\dot{x}$; id eſt ut $x^x + x^x lx + cx^{c-1} - yx^{y-1}$ ad $x^y lx + 1$, ita $y$ ad

“ quartam, erit hæc quarta Subtangens quæſita curvæ propoſitæ. Nota
“ interim quod multitudo terminorum contractionem plane non impe-
“ diat; cum enim mediante Logarithmica ſingulis ſeorſim aſſignari poſſit
“ quantitas æqualis, patet utique quantitates iſtas hoc modo aſſignatas
“ & ſimul ſumptas formare illam ipſam quæ quæritur.

“ Exempla hucuſque allata abunde illuſtrabunt proceſſum inſtituen-
“ dum in aliis primi gradus exponentialibus, exque illis haud difficul-
“ ter perſpicitur, quomodo Methodus applicanda ſit ad altiores gradus;
“ in omnibus enim par operandi ratio Obſervanda eſt; hucuſq; D. *Johannes Bernoulli.*

*Cheynæus* Analytice quidem & ex Natura Logarithmorum petita demonſtratione fundamentum harum Methodorum oſtendit uti pag. 14. Meth. Inverſ. videre eſt; et Fluentes Fluxionum Logarithmicalium ulterius longe provexit. Quod priuſquam aggrediamur, fluxiones hujuſmodi quantitatum quomodo ulterius accipiendæ ſunt ibidem ſcil. pag. 14 & 28 oſtendit, uti ſynopſice ſequitur.

$$
\phi : \begin{cases}
lx = \dfrac{\dot{x}}{x} \\[2ex]
lx + 1 = \dfrac{\dot{x}}{x+1} \\[2ex]
l : lx = x^{-1}l^{-1}x\dot{x} = \dfrac{\dot{x}}{xlx}. \\[2ex]
l^{n}x = nx^{-1}l^{n-1}x\dot{x} \\
x^{m}l^{n} : lx = mx^{m-1}l^{n} : lx\dot{x} + nx^{m-1}l^{n-1} : lxl^{-1}x\dot{x}. \\
x^{m}l^{n}xl^{r} : lx + a^{m+n+r} = \begin{cases} mx^{m-1}l^{n}xl^{r} : lx\dot{x} + \\ nx^{m-1}l^{n-1}xl^{r} : lx\dot{x} + \\ rx^{m-1}l^{n-1}xl^{r-1} : lx\dot{x} \text{ \&c. \&c.} \end{cases}
\end{cases}
$$

Vel fluxionibus hiſce datis eſſe illas Fluentes Correſpondentes.

Hiſce ita ſe habentibus, Fluentes Fluxionum Logarithmicarum in Caſibus generalioribus exhibuit *Cheyneus*; cujus rei ulteriorem explicationem (quia nonnullis obſcuriores hujuſmodi quoque Series viſæ ſunt) hic ſiſtere in Tyronum gratiam non gravabimur; præmiſſa imprimis

Regula *Cheynæana* pro Aſſumptionibus in Infinitum quæ hujuſmodi eſt $\overline{\overline{\overline{x^{m+1}+lx+l:lx+l:.l:lx+1}|^{n+1}}|^{p+1}}|^{q+1}$ &c. quæ Evoluta (inquit) & Coefficientibus Indeterminatis adfecta dabit Seriem aſſumendam pro Fluentibus Logarithmicis quæſitis. Et hinc patet progreſſio Aſſumptionum ad Infinitum quod ex Infra Explicandis Elucidatius patebit. Adnotato, interim, quod cum Aſſumptio ad terminos aliquot quorum Indices progreſſione arithmetica ab o ad infinitum excurrere poſſint, ſolum porrigatur, ut Fluens generaliſſime exhibeatur, Indicibus indeterminatis progreſſione indefinita excurrentibus, exhiberi omnino debet; & ex aſſumptione primaria (ut jamjam antea diximus prout Natura fluxionis exigit) cæteræ etiam infinities in infinitum excurrentes ſeſe facillime explicant; uti fuſius infra patebit. Jam 1°. Sit Fluxio data $x^m lx\dot{x}$ (eadem ſcil. cum *Cheynæana* pag. 14) Jam ſecundum regulam ſupra allatam aſſumenda erit $x^{m+1}\times\overline{lx+1}|$ Coefficientibus Indeterminatis adfecta *viz.* $Bx^{m+1}lx+Cx^{m+1}$ quod ex Natura Methodi directæ ſic ulterius eluceſcat; Aſſumatur primario $Bx^{m+1}lx$; cujus accepta fluxione habebis $\overline{m+1}\times Bx^m lx\dot{x}+(Bx^{m+1}\frac{\dot{x}}{x}=)\,Bx^m\dot{x}$; jam primum membrum cum fluxione data Comparabitur, & facta Comparatione, determinataque Coefficiente $B=\frac{1}{m+1}$, reſtabit altera fluxionis pars ſcil. $Bx^m\dot{x}$, pro qua eapropter aſſumenda erit Fluens $Cx^{m+1}$ & ſic aſſumptio terminatur ſeu prorſus exhauritur, & ubi aſſumptio primaria Series eſt Infinita, dabitur (pro Natura fluxionis) Series Infinita fluxionum reſtantium, pro quibus alia rurſus Series Infinita aſſumenda eſt, & ſic porro donec aut in Series Plures aut Infinitas excurrat Fluens. Et hoc ex intima rei Natura profluit, eo, quo ſupra oſtenſum eſt, modo; ex aſſumptione autem primaria & ejuſdem Fluxione cæteræ omnes ad Infinitum dependent. Sic in *Cheynæo* pro fluxione $x^m l^n x\dot{x}$ ſola aſſumptio Primaria $Ax^{m+1}l^n x$ dabit ſubſequentes omnes Terminos & Series.

1°. Fluxio $rx^m l^n x\dot{x}$ in unica Fluentis Serie abſoluta erit, & Fluentem dabit $\frac{r}{m+1}x^{m+1}l^n x-\frac{nr}{\overline{m+1}|^2}x^{m+1}l^{n-1}x$ &c. ad Inf. uti ex aſſumpta fluxione & cum data &c. Comparatione patebit; ubi ſi $n =$ ſit numero integro aut affirmativo aut etiam nihilo in fluxione quavis particulari quæ ſub hac formula cadit Series abrumpetur, & Fluens exhibebitur in terminis finitis.

2°. Si Fluxio data eſſet $x^m l : lx\dot{x}$, aſſumenda erit $x^{m+1} \times \overline{l : lx}$ evoluta &c. quæ in hujuſmodi evolutione devenit $l^{-1}x + l^{-2}x + l^{-3}x$ &c. ad Inf. Aſſumenda ergo erit $x^{m+1} \times \overline{l : lx + l^{-1}x}$ &c. ad Inf. Coefficientibus Indeterminatis adfecta ſcil. $Ax^{m+1} l ; lx \ldots Bx^{m+1} l^{-1} x \ldots Cx^{m+1} l^{-2} x$ &c.

Sit jam 3°. Fluxio data $x^m l^2 : lx\dot{x}$; hujus aſſumpta Fluens Primaria ſcil. $Ax^{m+1} l^2 : lx \ldots Bx^{m+1} l^{-1} xl : lx \ldots Cx^{m+1} l^{-2} xl : lx$ &c. relinquet pro Serie Secunda aſſumenda hanc Fluxionum Seriem reſtantem viz. $x^m l^{-2} \dot{x}x \ldots x^m l^{-3} \dot{x}x$ &c. pro quibus rurſus aſſumenda erit $x^{m+1} l^{-2} x \ldots x^{m+1} \times l^{-3} x$ &c. ad Inf. Coefficientibus Indeterminatis adfecta, & Series exhaurietur, ſeu nullæ aliæ invenientur Fluxiones reſtantes &c.

Rurſus ſi eſſet Fluxio data $x^m l^3 : lx\dot{x}$ ex aſſumptione primaria (Coefficientibus Indeterminatis adficienda)

ſcil. $x^{m+1} l^3 : lx \ldots l^{-1} xl^2 : lx \ldots l^{-2} xl^2 : lx \ldots l^{-3} xl^2 : lx$ &c.
provenirent $\ldots\ldots\ldots l^{-2} xl : lx \ldots\ldots l^{-3} xl : lx \ldots l^{-4} l : lx$ &c.
$\ldots\ldots\ldots l^{-3} x \ldots\ldots\ldots l^{-4} x \ldots\ldots\ldots l^{-5} x$ &c.

Coefficientibus pariter Indeterminatis adficiendæ, & ſic exhauriretur; & ſic porro. Unde in eo genere Univerſaliter ſi data eſſet fluxio $x^m l^n : lx\dot{x}$; aſſumenda foret

$$x^{m+1} l^n : lx \ldots x^{m+1} l^{-1} xl^{n-1} : lx \ldots x^{m+1} l^{-2} xl^{n-1} : lx \text{ \&c.}$$
$$x^{m+1} l^{-2} xl^{n-2} : lx \ldots x^{m+1} l^{-3} xl^{n-2} : lx \text{ \&c.}$$

&c. ad Inf. &c. ad Inf.

Coefficientibus Indeterminatis adficienda ſcil.

$$\alpha x^{m+1}l^{n}:lx\,..\,Ax^{m+1}l^{-1}xl^{n-1}:lx\,..\,Bx^{m+1}l^{-2}xl^{n-1}:lx\,..\,Cx^{m+1}l^{-3}xl^{n-1}:lx \text{ \&c.}$$

$$Dx^{m+1}l^{-2}xl^{n-2}:lx\,..\,Ex^{m+1}l^{-3}xl^{n-2}:lx\,..\,Fx^{m+1}l^{-4}xl^{n-2}:lx \text{ \&c.}$$

&c. &c. &c.

& acceptis fluxionibus, habebis $x^{m}l^{n}:lx\dot{x} = \overline{m+1}\,\alpha x^{m}l^{n}:lx\dot{x}$. Unde $1°.\ 1 = \overline{m+1}\,\alpha$ & $\frac{1}{\overline{m+1}|} = \alpha$ &c. Et pergendo habebis pro reliqua Serie Comparationum

$$\left.\begin{array}{r} n\alpha x^{m}l^{-1}xl^{n-1}:lx\dot{x} \\ m+1Ax^{m}l^{-1}xl^{n-1}:lx\dot{x} \end{array}\right\} \left.\begin{array}{l} -1Ax^{m}l^{-2}xl^{n-1}:lx\dot{x} \\ m+1B\ldots\ldots\ldots\ldots \end{array}\right\} \begin{array}{l} -2Bx^{m}l^{-3}xl^{n-1}:lx\dot{x} \\ m+1C\ldots\ldots\ldots\ldots \end{array}$$

$$\left.\begin{array}{r} -3Cx^{m}l^{-4}xl^{n-1}:lx\dot{x} \\ m+1D\ldots\ldots\ldots\ldots \end{array}\right\} \text{\&c. ad Infinitum. Unde provenirent}$$

$2°.\ n\alpha = m+1A.\quad A = \frac{n\alpha}{m+1} = \frac{nr}{\overline{m+1}|^{2}}.$

$3°.\ -1A = m+1B\,.\,B = -\frac{1A}{m+1} = -\frac{1nr}{\overline{m+1}|^{3}}.$

$4°.\ -2B = m+1D\,.\,C = -\frac{2B}{m+1} = -\frac{1\times 2nr}{\overline{m+4}|^{4}}.$

$5°.\ -3C = m+1D\,.\,D = -\frac{3C}{m+1} = -\frac{1\times 2\times 3nr}{\overline{m+1}|^{5}}.$

$6°.\ -4D = m+1E.$ Unde $E = -\frac{4D}{m+1} = -\frac{1\times 2\times 3\times 4nr}{\overline{m+1}|^{6}}.$

Et ita porro ad Infinitum. Et pro Comparatione ulteriori reſtarent

$$\overline{n-1}Ax^{m}l^{-2}xl^{n-2}:lx\dot{x}\ldots\ldots\overline{n-1}Bx^{m}l^{-3}xl^{n-2}:lx\dot{x}\ldots\ldots$$

$$\overline{n-1}Cx^{m}l^{-4}xl^{n-2}:lx\dot{x}\ldots\ldots\overline{n-1}Dx^{m}l^{-5}xl^{n-2}:lx\dot{x} \text{ \&c.}$$

Pro quibus aſſumenda eſſet $Ex^{m+1}l^{-2}xl^{n-2}:lx\ldots Fx^{m+1}l^{-3}xl^{n-2}:lx\ldots$ $Gx^{m+1}l^{-4}xl^{n-2}:lx$ &c. Cujus, accepta fluxione, proveniret altera reſtantium Fluxionum Series viz.

$$\left.\begin{array}{l} m+1Ex^{m}l^{-2}xl^{n-2}x\dot{x}-2Ex^{m}l^{-3}xl^{n-2}:lx\dot{x} \\ \qquad\qquad m+1F\ldots\ldots\ldots\ldots \end{array}\right\} \begin{array}{l} +\overline{n-2}\times Ex^{m}l^{-3}xl^{n-3}:lx\dot{x}\ \&c. \\ \qquad\qquad \&c. \end{array}$$

quæ ultra reſtaret pro Serie adhuc ſubſequente, & ſic ad Infinitum: & habebis jam $n-1A=m+1E$. $E=\frac{n-1A}{m+1}$. $-2E+mF=\overline{n-1}B$. ſeu $m+1F=\overline{n-1}B+2E$. & $F=\frac{\overline{n-1}B+2E}{m+1}$ &c. quæ cum *Cheynæanis* Coincidunt. Et hinc patet Origo Naturalis (hoc in Caſu) Serierum Infinities-Infinitiarum.

Sit rurſus Fluxio data $px^{m}l^{r}xl^{n}:lx\dot{x}$ Exemplum ſcil. *Cheynean.* pag. 32. Aſſumatur (ſeu fiat aſſumptionis primariæ terminus primus)
1°. $\alpha x^{m+1}l^{r}xl^{n}:lx$ erit ejuſdem fluxio
$\overline{m+1}\alpha x^{m}l^{r}xl^{n}:lx\dot{x}+r\alpha x^{m}l^{r-1}xl^{n}:lx\dot{x}+n\alpha x^{m}l^{r-1}xl^{n-1}:lx\dot{x}$ erit itaque $px^{m}l^{r}xl^{n}:lx\dot{x}=\overline{m+1}\alpha x^{m}l^{r}xl^{n}:lx\dot{x}$ : Unde $p=\overline{m+1}\alpha$. $\alpha=\frac{p}{m+1|}$.

Sed quia jam reſtant $r\alpha x^{m}l^{r-1}xl^{n}:lx\dot{x}+n\alpha x^{m}l^{r-1}xl^{n-1}:lx\dot{x}$.

2°. Aſſumatur $Ax^{m+1}l^{r-1}xl^{n}:lx=$ ſecundo termino Fluentis; cujus Fluxio erit $m+1Ax^{m}l^{r-1}xl^{n}:lx\dot{x}-\overline{r-1}Ax^{m}l^{r-2}xl^{n}:lx\dot{x}+nAx^{m}l^{r-1}xl^{n-1}:lx\dot{x}$. Et Comparatio proxima erit $m+1Ax^{m}l^{r-1}xl^{n}:lx\dot{x}=r\alpha x^{m}l^{r-1}xl^{n}:lx\dot{x}$.

Seu $m+1A=r\alpha$. & $A=\frac{r\alpha}{m+1}=\frac{rp}{m+1|^{2}}$.
Unde $Ax^{m+1}l^{r-1}xl^{n}:lx=\frac{rp}{m+1|^{2}}x^{m+1}l^{r-1}xl^{n}:lx$.

3°. Jam pro proximo fluxionis reſtantis termino aſſumatur $Bx^{m+1}l^{r-2}xl^{n}:lx$; & ejuſdem Fluxio erit
$\overline{m+1}Bx^{m}l^{r-2}xl^{n}:lx\dot{x}+\overline{r-2}Bx^{m}l^{r-3}xl^{n}:lx\dot{x}+nBx^{m}l^{r-2}xl^{n-1}:lx\dot{x}$.

Et $-\overline{r-1}Ax^m l^{r-2}xl^n : lx\dot{x} = \overline{m+1}Bx^m l^{r-2}xl^n : lx\dot{x}$

Et $-\overline{r-1}A = \overline{m+1}B. \ -\frac{\overline{r-1}A}{m+1} = B = \frac{\overline{r-1}\times r\times p}{\overline{m+1}|^3}$.

4°. Aſſumatur $Cx^{m+1}l^{r-3}xl^n : lx$ cujus Fluxio erit

$$\overline{m+1}Cx^m l^{r-3}xl^n : lx\dot{x} + |\underline{\overline{r-3}Cx^m l^{r-4}xl^n : lx\dot{x} + nCx^m l^{r-3}xl^{n-1} : lx\dot{x}}$$

Erit $\overline{r-2}B = \overline{m+1}C.$ & $C = \frac{\overline{r-2}B}{m+1} = \frac{\overline{r-2}\times\overline{r-1}\times r\times p}{\overline{m+1}|^4}$.

5°. Aſſumatur $Dx^{m+1}l^{r-4}xl^n : lx.$ & habebis

$$\overline{m+1}Dx^m l^{r-4}xl^n : lx\dot{x}|\underline{+\overline{r-4}Dx^m l^{r-1}xl^n : lx\dot{x} + nDx^m l^{r-4}xl^{n-1} : lx\dot{x}}$$

Erit $\overline{r-3}C = \overline{m+1}D. \ \frac{\overline{r-3}C}{m+1} = D = \frac{\overline{r-3}\times\overline{r-2}\times\overline{r-1}\times r\times p}{\overline{m+1}|^5}$. &c.

ad Inf. pro prima Serie; unde e reſtantibus tertiis ex aſſumptione Primaria exurget Series Fluxionum quæ dabit ſeu determinabit Seriem Aſſumendam Secundam; & ſic porro ad Infinitum, viz. pro

$n\alpha x^m l^{r-1}xl^{n-1} : lx\dot{x} \ldots nAx^m l^{r-1}xl^{n-1} : lx\dot{x} \ldots nBx^m l^{r-2}xl^{n-1} : lx\dot{x} \ldots\ldots$

$\ldots nCx^m l^{r-3}xl^{n-1} : lx\dot{x} \ldots\ldots nDx^m l^{r-4}xl^{n-1} : lx\dot{x}$ &c. Aſſumenda foret

$Gx^{m+1}l^{r-1}xl^{n-1} : lx + Hx^{m+1}\times l^{r-2}xl^{n-1} : lx$ &c. ad Infinitum quæ adhuc

ſuam reſtantium Seriem relicturæ ſunt, & ſic porro ad Inf. ſeu poſterioris hujus æquationis accepta Fluxio Comparanda erit cum reſpectiva reſtantium Serie, & Series alia adhuc reſtabit, viz.

$\varphi : Gx^{m+1}l^{r-1} : xl^{n-1} : lx = \overline{m+1}Gx^m l^{r-1}xl^{n-1} : lx\dot{x} \ldots\ldots\ldots\ldots\ldots$

$\ldots\ldots \overline{r-1}Gx^m l^{r-2}xl^{n-1} : lx\dot{x} \ldots\ldots \overline{n-1}Gx^m l^{r-2}xl^{n-2} : lx\dot{x}$ jamq; haberes

$n\alpha = \overline{m+1}G.$ ſeu $G = \frac{n\alpha}{m+1} = \frac{np}{\overline{m+1}|^2}$, & ſic porro ad Inf.

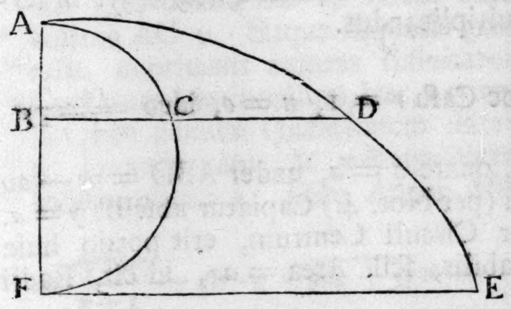
Haſce applicat *Cheynæus* ad Fluxionum Logarithmo-logarithmicarum Fluentes obtinendas; & uſum oſtendit in Exponentialibus ſecundi gradus. Uti videre eſt Pag. 34. *Meth. Inverſ.* quam conſulat Lector. Si jam alii Irrationalitatis gradus adeſſent, Series hæ Infinities Infinitæ in unam partitionem Colligendæ eſſent, ubi unaquæque unam lineam conficeret, & ſic protenderentur ad Inf. & ſecunda eodem modo protenſa alium Infinitatis gradum ſeu gradum alium Infinities-infinitum compleret, ſeu duos gradus conficeret; Tertia tertium daret & ſic porro gradus Infinito-Infinities Infiniti ad Infinitum facillime (modo literæ ſuppeterent) Exprimerentur.

CAP.

# CAP. VIII.

## *De Quadratura Curvarum Irrationalium Craigiana*, &c.

DOM. *J. Craigius* poſt Editas diu antea Curvarum Quadraturas quaſdam, tandem ad figuras quas vocat Irrationales pervenit; harum Specimina e Tranſact. Philoſ. Nº 232. Verbis inſequentibus diſſerit; “ Sit ACF Semicirculus cujus diameter eſt AF, ADE Curva Geometrice Irrationalis, cujus ordinatim applicata BD, ſecat Semicirculum in C Quantitates vero ſic deſignentur; Diameter AF $= 2a$, abſciſſa AB $= y$, Arcus AC $= v$, Ordinata BD $= z$: Sitque $ruy^{n}$ æquatio generalis exprimens naturas Curvarum Geometrice Irrationalium ADE, in qua $r$ denotat quantitatem quamlibet datam & determinatam, & $n$ Exponentem Indefinitum quantitatis Indeterminatæ $y$. Dico Aream ABD $= \frac{ruy^{n+1}}{n+1} - qv$



$$+ \sqrt{2ay - yy} \times \frac{rx}{\overline{n+1}|^{2}}\, y^{n} + \frac{2nra^{2} + ra^{2}}{n \times \overline{n+1}|^{2}}\, y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1}\, y^{n-2}$$

$+ aB$

“ $+ \frac{aB \times \overline{2n-3}^{2}}{n-2} y^{n-2} + \frac{aC \times \overline{2n-5}}{n-3} y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4} y^{n-5}$

“ $+ \frac{aE \times \overline{2n-9}}{n-5} y^{n-6} +$, &c. De hac Serie Infinita hæc ſunt notanda,

“ (1.) Quod Literæ Majuſculæ A, B, C, D, E, &c. deſignent Coeffici-
“ entes Terminorum ipſis immediate præcedentium, ſcil.

$A = \frac{2nra^2 + ra^2}{n \times \overline{n+1} \times \overline{n-1}}$, $B = \frac{aA \times \overline{2n-1}}{n-1}$, $C = \frac{aB \times \overline{2n-3}}{n-2}$, & ſic

“ porro. (2.) Quod ſi exponens $n$ ſi Numerus Integer & poſitivus,
“ aut nihilo æqualis, vel etiam ſi $2n$ ſit Numerus Impar, tum Quadra-
“ tura Spatii ABD exhibeatur per Quantitatum finitam; Serie in his
“ Caſibus abrumpente. (3 ) Quod $q$ deſignet terminum ultimo abrum-
“ pentem. (4.) Quod omnes illæ Figuræ in quibus Series abrumpitur,
“ habeant unam Portionem Geometrice quadrabilem, ex ipſa Serie fa-
“ cillime aſſignabilem : nimirum ſi capiatur abſciſſa $y = \overline{r}|^{\frac{1}{n+1}}$
“ $\times \overline{nq+q}|^{\frac{1}{n+1}}$; erit huic abſciſſæ Competens Area Geometrice
“ quadrabilis. (5.) Quod ſolus terminus Irrationalis $\sqrt{\overline{2ay-yy}|}$ in ter-
“ minatos ipſum ſequentes ſit Multiplicandus.

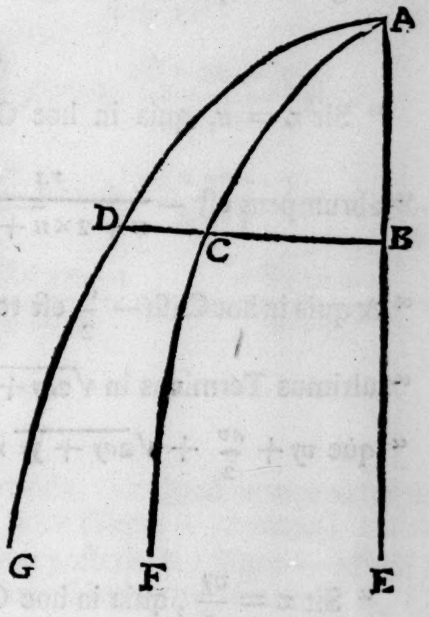
“ *Ex.* 1. Sit $z = v$, quia in hoc Caſu $r = 1$, $n = o$, ideo $\frac{ra}{\overline{n+1}|^{2}} y^n$
“ eſt terminus ultimo abrumpens, quare $q = a$, under $ABD = vy - av$
“ $+ a + \sqrt{2ay-yy}$: & proinde ſi (per Not. 4.) Capiatur abſciſſa $y = a$,
“ id eſt, ſi Ordinata tranſeat per Circuli Centrum, erit portio huic
“ Competens Geometrice quadrabilis, ſcil. Area $= aa$, id eſt, Radii
“ Quadrato.

“ *Ex.* 2. Sit $z = \frac{vy}{a}$ quia in hoc caſu $r = \frac{a}{1}$, $n = 1$, ideo
“ $\frac{anraa + raa}{n \times \overline{n+1}|^{2}} y^{n-1}$ eſt terminus ultimo abrumpens, quare $q = \frac{3a}{4}$, unde
“ $ABD = \frac{vy^2}{2a} - \frac{3av}{4} + \frac{y+3a}{4} \sqrt{2ay-y}$, & proinde ſi ( per Not. 4)

“ capiatur $y=\sqrt{\frac{3aa}{2}}$, erit huic abſciſſæ competens Area Geometrice
“ Quadrabilis, ſcil. Area $=\sqrt{\overline{\sqrt{6a^4}-\frac{3a^2}{2}}}\times\frac{\sqrt{3a^2}}{32}+\frac{3a}{4}$.

“ *Ex.* 3. Sit $z=\frac{vy^2}{aa}$; in hoc caſu $r=\frac{1}{aa}$, $n=2$, ideo $\frac{aA\times 2n-1}{n-1}y^{n-2}$
“ eſt terminus ultimo abrumpens, ergo $q=\frac{5a}{6}$; unde per Seriem In-
“ finitam erit ABD $=\frac{6vy^3-15a^3v+\overline{2ay^2+5a^2y+15a^3}\sqrt{2ay-yy}}{18a^2}$ &
“ proinde ſi (per Not. 4.) capiatur $y=\sqrt[3]{\frac{5a^3}{2}}$ erit Abſciſſæ competens
“ Area Geometrice Quadrabilis, ſcil. Area $=\frac{2ay^2+5a^2y+15a^3}{18a}$
“ $\times\sqrt{2ay-yy}$.

“ Secundo, Sit ACB Parabola, cujus Axis AE, Vertex A, & Latus
“ Rectum BA.



“ Sitque ADG Curva Geometrice Irra-
“ tionalis, cujus Ordinata BD ſecat Pa-
“ rabolam in C. Et vocatur Abſciſſa
“ AB $y$, Ordinata BD $z$, Arcus Para-
“ bolicus AC $v$. Sitque Æquatio gene-
“ ralis, exprimens naturas Infinitarum
“ Curvarum Irrationalium, hæc, $z=rvy^n$
“ in qua $r$ denotat Quantitatem datam
“ & determinatam, & $n$ Exponentem
“ Indeterminatæ $y$. Dico Aream ABD

“ $=\frac{ry^{n+1}v}{n+1}-qv+\sqrt{2ay+yy}\times-$

“ $\frac{r}{n+2\times n+1}y^{n+1}-\frac{ra}{n+2\times\overline{n+1}|^2}y^n$

“ $+ \frac{raa \times 2n+1}{n \times n+2 \times \overline{n+1}|^2} y^{n-1} - \frac{aA \times 2n-1}{n-1} y^{n-2} + \frac{aB \times 2n-3}{n-2} y^{n-3}$

“ $\frac{aC \times 2n-5}{n-1} y^{n-4} +$, &c De hac Serie hæc ſunt notanda ; 1. Quod
“ Literæ majuſculæ A, B, C, &c. denotant Coefficientes terminorum
“ ipſis præcedentium. 2. Quod ſi exponens $n$ ſit integer poſitivus aut
“ nihilo æqualis, aut etiam ſi $2n$ ſit numerus Impar, tum Quadratura
“ exhibeatur per numerum Terminorum finitum ; Serie in his Caſibus
“ abrumpente.

3. “ Quod $+ q$ ſit æqualis Termino ultimo abrumpenti.

4. “ Quod ex terminis Quantitatem $\sqrt{2ay+yy}$ multiplicantibus ultimo
“ abrumpens ſit duplicandus.

5. “ Quod omnes illæ Figuræ in quibus $n$ eſt numerus Integer poſiti-
“ vus & impar ; vel generalius, omnes illæ Figuræ, in quibus ulti-
“ mus Terminus abrumpens habet Signum Affirmativum ſeu $+$, ha-
“ beant unam Portionem Geometrice Quadrabilem, & ex ipſa Serie facile
“ aſſignabilem, ſumendo Abſciſſam ut in Not. 4. præcedentis Seriei.

*Exemp.* 1.

“ Sit $z = v$, quia in hoc Caſu $r = 1$, $n = 0$, ideo terminus ultimo
“ abrumpens eſt $- \frac{ra}{n+2 \times \overline{n+1}|^2} y^n$, unde $+ q = - \frac{a}{2}$ (per Not. 3.)
“ & quia in hoc Caſu $- \frac{a}{2}$ eſt terminus ultimo abrumpens, ideo $-a$ eſt
“ ultimus Terminus in $\sqrt{2ay+yy}$ multiplicandus (per Not. 4.) adeo-
“ que $vy + \frac{av}{2} + \sqrt{2ay+yy} \times \overline{-\frac{1}{2}y-a}. =$ Areæ.

*Exemp.* 2.

“ Sit $z = \frac{vy}{a}$, quia in hoc Caſu $r = \frac{1}{a}$, $n = 1$, ideo terminus ulti-
“ mo abrumpens eſt $\frac{raa \times 2n+1}{n \times n+2 \times \overline{n+1}|^2} y^{n-1} = \frac{a}{4}$, unde $q = \frac{a}{4}$, $\frac{a}{2}$ ulti-

“ mus terminus in $\sqrt{2ay+yy}$ multiplicandus; adeoque ABD $= \frac{vyy}{2a}$
“ $-\frac{av}{4} + \sqrt{2ay+y^2} \times -\frac{y}{6a} - \frac{y}{12} + \frac{a}{2}$; & ſi capiatur $y = \sqrt{\frac{aa}{2}}$,
“ erit Area competens Grometrice Quadrabilis, ſcil. Area $= \frac{1}{12} \sqrt{:\sqrt{2a^4+\frac{a^2}{2}}}$
“ $\times 5a - \sqrt{\frac{a^2}{2}}$.

“ 3. Sit ACF Semicirculus; ADE Curva Geometrice Irrationalis, cujus Ordinata BD ſecat Semicirculum in C;
“ Quantitates vero deſignentur ut prius, ſcil. Diam.
“ AF $= 2a$, Abſciſſa AB $= y$, Arcus AC $= v$, Ordinata BD $= z$.
“ Sitque $z = rv^2y^n$ Æquatio exprimens naturas Curvarum ADE; in
“ qua $r$ denotat Quantitatem quamlibet datam & determinatam, & $n$
“ exponentem Indefinitum Quantitatis Indeterminatæ $y$. Dico Aream

Tranſact. N° 235. *Vid. Fig.* 1.

$$“\ \mathrm{ABD} = \frac{rv^2y^{n+1}}{n+1} - qv^2 + v\sqrt{2ay-yy} \times \frac{2ra}{\overline{n+1}|^2}y^n$$

$$“\ + \frac{2ra^2 \times \overline{2n+1}}{n \times \overline{n+1}|^2}y^{n-1} + \frac{aA \times \overline{2n-1}}{n-1}y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2}y^{n-3}$$

$$“\ + \frac{aC \times \overline{2n-5}}{n-3}y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4}y^{n-5} + \frac{aE \times \overline{2n-9}}{n-5}y^{n-6}, \text{\&c.}$$

$$“\ - \frac{2ra^2}{\overline{n+1}|^3}y^{n+1} - \frac{2ra^3 \times \overline{2n+1}}{n^2 \times \overline{n+1}|^2}y^n - \frac{a^2A \times \overline{2n-1}}{\overline{n-1}|^2}y^{n-1} - \frac{a^2B \times \overline{2n-3}}{\overline{n-2}|^2}y^{n-2}$$

$$“\ - \frac{aC \times \overline{2n-5}}{\overline{n-3}|^2}y^{n-3}, \text{\&c.}$$

“ De hoc Theoremate hæc ſunt notanda. 1. Quod componatur ex
“ duabus Seriebus Infinitis; quarum prior (Signo + connexa) multi-
“ plicatur in $v\sqrt{2ay-yy}$; termini autem poſterioris (Signo — affecti)
“ ſunt abſoluti. 2. Quod in priori Serie Literæ majuſculæ A, B, C,
“ D, E, &c. deſignent Coefficientes terminorum ipſis reſpective præce-
“ dentium; nec non in poſteriori eoſdem obtineant Valores, quos in
“ pri-

" priori. 3 Quod Quadratura exhibeatur per Quantitatem finitam,
" quando $n$ eſt Numerus Integer poſitivus, aut nihilo æqualis, vel etiam
" ſi $2n$ ſit Numerus Impar: Nam in his Caſibus utraque Series ab-
" rumpetur. 4. Quod $2q$ ſit æqualis ultimo termino abrumpenti pri-
" oris Seriei.

*Exemp.* 1.

" Sit $z = \frac{v^2}{a}$, quia in hoc caſu $n = 0$. $r = \frac{1}{a}$, ideo erit Area
" $= \frac{yv^2}{a} - v^2 + 2v\sqrt{2ay - yy} - 2ay$.

*Coroll.* " Integra Figura eſt æqualis duplo Quadrato, cujus Latus eſt
" ACF, dempto Diametri Quadrato.

*Exemp.* 2.

" Sit $z = \frac{yv^2}{a^2}$, quia in hoc caſu, $n = 1$, $r = \frac{1}{a^2}$, ideo erit Area
" ABD $= \frac{y^2v^2}{2a^2} - \frac{1}{4}v^2 + v\sqrt{2ay - yy} \times \overline{\frac{v}{2a} \times \frac{3}{2}} - \frac{1}{4}y^2 - \frac{3ay}{2}$.

*Exemp.* 3.

" Sit $z = \frac{y^2v^2}{a^3}$, quoniam in hoc caſu $n = 2$, $r = \frac{1}{a^3}$, ideo erit Area
" ABD $= \frac{y^3v^2}{3a^3} - \frac{5}{6}v^2 + v\sqrt{2ay - y^2} \times \overline{\frac{2y^2}{9aa} + \frac{5y}{9a} + \frac{5}{3}} - \frac{2y^3}{27a} - \frac{5y^2}{18}$
" $- \frac{5ay}{3}$.

Hæcce autem omnia ad Infinitum pergentia, in uno generali Theoremate, concludit Dr. *Cheyneus*, quod hic explicare Operæ pretium duximus,

ſcil.

Sit jam Fluxio data generalis illa *Cheynaana*, ſcil. $\dot{y} \times ry^n \times \mathrm{F}: \overline{\frac{ay}{\sqrt{2ay+y^2}}}\Big|^m$ vel $ry^n v^m \dot{y}$, & ex Serie prima (more *Craigiano*) debite aſſumpta Coefficientibus Indeterminatis adfecta, reſtabit Series Fluxionum ſecunda ſeu terminorum involventium $v^{m-2}$ pro quibus aſſumenda erit ſecunda Fluens e qua (ſumptis Fluxionibus) rurſus reſtabit Fluxionum Series tertia $v^{m-3}$ involventium, & ſic ad Infinitum, adeo ut Fluens erit.

$$1.\ \frac{rv^m y^{n+1}}{n+1} - bv^m + \sqrt{2ay-yy} \times \frac{mra}{\overline{n+1}|^2} y^n v^{m-1}$$

$$+ \frac{\mathrm{A}a \times \overline{2n+1}}{n} y^{n-1} v^{m-1}, \text{ \&c. ad Inf.}$$

$$2.\ \frac{m \times \overline{m-1} \times ra^2}{\overline{n+1}|^3} y^{n+1} v^{m-2} - \frac{\overline{m-1}\,\mathrm{A}a^2 \times \overline{2n+1}}{n^2} y^n v^{m-2}, \text{ \&c.}$$

ad Inf. &c. + $cv^{m-2}$.

$$3.\ \frac{-m \times \overline{m-1} \times \overline{m-2} \times ra^3}{\overline{n+1}|} y^n v^{m-3} -$$

$$\frac{\overline{m-1} \times \overline{m-2}\,\mathrm{A}a^3 - n^2 \mathrm{E}a \times 2n+1}{n^3} y^{n-1} v^{m-3} \times \sqrt{2ay-yy}, \text{ \&c.}$$

$$4.\ \frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}\,ra^4}{\overline{n+1}|^5} y^n v^{m-4}$$

$$+ \frac{\overline{\overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \mathrm{A}a^4 + \overline{m-3} \times n^2 \mathrm{E}a^2 \times 2n+1}}{n^4} y^{n-1} v^{m-4}$$

+ &c. — $dv^{m-4}$.

$$5.\ \frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}\,ra^5}{\overline{n+1}|^6} y^n v^{m-5}, \text{ \&c.}$$

$\times \sqrt{2ay-yy}$, &c. &c. ad Inf.

## *Obſervanda in hoc Theoremate proponit* Cheynæus.

1. “ QUOD conſtet infinitis Numero Seriebus infinitis; quarum “ prior habet omnes terminos præter ſecundum Signo + “ adfectos, Binæ ſequentes habent omnes terminos Signo — adfectos, “ Binæ inde ſequentes Signo + ſic in ſequentibus per binas alternatim “ præter unicum ſemper terminum in Seriebus paribus.

2. “ Quod in ſingularibus iſtis Terminis jam memoratis Serierum “ parium, & in prima Serie, ſcil. in Termino ſecundo Seriei primæ “ $bv^m$, eſt $mb =$ ultimo Termino abrumpenti iſtius Seriei; & $cv^{m-2}$ “ Seriei ſecundæ, eſt $m - c =$ ultimo Termino abrumpenti Seriei “ tertiæ; in $dv^{m-1}$ Seriei quartæ eſt $\overline{m - 2d} =$ ultimo termino abrumpenti Seriei quintæ, & ſic in cæteris.

3. “ Quod Series particulares ſunt alternatim abſolutæ & multi- “ plicatæ in $\sqrt{2ay - yy}$, ſcil. prima Series (præter binos primos Termi- “ nos) ducitur in $\sqrt{2ay - yy}$; ſecunda eſt abſoluta; tertia ducitur in “ $\sqrt{2ay - yy}$; quarta eſt abſoluta, & ſic deinceps.

4. “ Quod Literæ majuſculæ eundem ubique valorem obtinent, & “ quod ubicunque emicat majuſcula nova in Serie aliqua, ſemper ſigni- “ ficat Integrum Coefficientem præcedentem iſtius Seriei; adeoque “ promptum erit oculi intuitu Series continuare *v. g.* ſecundus Termi- “ nus ſeptimæ Seriei erit

$$\frac{-\overline{m-1}\times\overline{m-2}\times\overline{m-3}\times\overline{m-4}\times\overline{m-5}\times\overline{m-6}\times Aa^7-\overline{m-3}\times\overline{m-4}\times\overline{m-5}}{n^6}$$

$$\underline{\times\, m-6\times n^2\times Ea^5-\overline{m-5}\times\overline{m-6}\times n^4\times Ia^3-n^6\times Na\times 2n+1}\; y^n v^{m-7}.$$

5. “ Quod Numerus Serierum finitus evadit quando *m* eſt Numero “ Integro & Affirmativo aut nihilo æqualis.

6. “ Quod Numerus terminorum in particularibus Seriebus finitus “ evadet, quando *n* Numero Integro & Affirmativo aut nihilo, vel $2n$ “ Numero Impari affirmativo aut nihilo æqualis eſt.

7. “ Quod denique niſi tum *m* tum *n* æquales fuerint numero alicui “ Integro & Affirmativo aut nihilo, vel $2n$ Numero Impari Affirmativo “ aut nihilo, fluens terminis Numero finitis non exhibebitur.

Sit jam in Exemplum Theorema præcedens *Craigianum*, viz. pro $rv\dot{y}^n$ & invenietur eſſe hujus caſus particularis. Namque erit $m=1$, & $n=n$, &

$$\frac{rv^my^{n+1}}{n+1}=\frac{rvy^{n+1}}{n+1}.-bv^m=-qv.\sqrt{\overline{2ay-yy}}\times\frac{mra}{\overline{n+1}|^2}=\sqrt{\overline{2ay-yy}}\times\frac{ra}{\overline{n+1}|^2}y^n$$

*Exempl.* Cheynæi *in Rudim.* p. 22.

( nam hic $v$ ob terminum $m-1=0$ evaneſcit) & ſic porro. Sic $v^3\dot{y}$ faciendo $m=3$, $n=1$, $r=1$, dat $\frac{1}{2}v^3y^2-bv^3+\sqrt{\overline{2ay-yy}}+\frac{3}{4}av^2y+\frac{9}{4}a^2v^2$ (hic abrumpitur in hac Serie ob $n-1=0$ in terminis ſequentibus) $\frac{3}{4}a^2y^2v-\frac{9}{2}a^3yv$ (hic etiam ſiſtendum in ſecunda Serie) $+Cv-\frac{3}{8}a^3y-\frac{27}{8}a^4$. Hic Fluens omnino terminatur, & quia $\frac{9}{4}a^2y^2$ eſt terminus ultimo abrumpens in prima Serie, igitur $3b=\frac{4}{9}a^2$ adeoque $b=\frac{3}{4}a^2$. Item $\frac{27}{8}a^4$ eſt terminus ultimo abrumpens in tertia Serie, ideo $c=\frac{27}{8}a^4$, quare vera Fluens eſt $\frac{1}{2}v^3y^2-\frac{3}{4}a^2v^3+\sqrt{\overline{2ay-yy}}\times\overline{\frac{3}{4}av^2y+\frac{9}{4}a^2v^2}$ $-\frac{3}{4}a^2y^2v-\frac{9}{3}a^3y^2+\frac{27}{8}a^4v-\overline{\frac{3}{8}a^3y-\frac{27}{8}a^4}\times\sqrt{\overline{2ay-yy}}$. Rurſus $v^3y^{\frac{1}{2}}\dot{y}$, ubi $m=3$, $n=1$, $r=1$, debite applicata dabit

$$\frac{2}{3}v^3y^{\frac{1}{2}}-bv^2+\sqrt{\overline{2ay-yy}}\times\overline{\frac{4}{3}ay^{\frac{1}{2}}v^2+\frac{16}{3}a^2y^{-\frac{1}{2}}v^2}-\frac{16}{9}a^2y^{\frac{3}{2}}v$$

$-\frac{64}{3}y^{\frac{1}{2}}v+Cv$, $-\frac{33}{27}a^3y^{\frac{1}{2}}-\frac{1024}{27}a^4y^{-\frac{1}{2}}$ ubi quia $\frac{16}{3}a^2$ eſt Coefficiens ultimi Termini abrumpentis in prima Serie $3b=\frac{16}{3}a^2$ adeoque $b=\frac{16}{3}a^2$, item $c=\frac{1024}{27}a^4$. Eodem modo $z=ry^{-n}v$ ( ubi $v$ denotat Arcum Circuli cujus Radius $a$, Ordinata $\sqrt{\overline{2ay-yy}}$) dabit Aream

$$=\frac{ry^{-n+1}v}{-n+1}+\sqrt{\overline{2ay-yy}}\times\frac{-r}{3-2n\times1-n}y^{-n+1}+\frac{\overline{n-2}\times-A}{5-2\times a}y^{-n+2}$$

$$+\frac{\overline{n-3}\times-B}{\overline{7-2n}\times a}y^{-n+3}+\frac{\overline{n-4}\times-C}{9-2n\times a}y^{-n+4}+\frac{\overline{n-5}\times-D}{11-2n\times a}y^{-n+5}$$

$$\frac{n-6+-E}{\overline{13-2n}\times a}y^{-n+6},\ \&c.$$

Sit jam Fluxio data $v^3\dot{x}$ unciatim inveſtiganda, ubi $v = \mathrm{F} : \frac{r\dot{x}}{\sqrt{2rx - x^2}}$.

Sit $\mathrm{A}v^3x =$ primo Termino. Erit $\varphi : v = \frac{r\dot{x}}{\sqrt{2rx - x^2}}$ & $\varphi : \mathrm{A}v^3x = \mathrm{A}v^3\dot{x}$ $+ 3\mathrm{A}\dot{v}^2 vx$.

$$\mathrm{A}v^3\dot{x} = v^3\dot{x}. \quad \mathrm{A} = 1$$

Reſtat $3\mathrm{A}^2\dot{v}x = \frac{3\mathrm{A}v^2rx\dot{x}}{\sqrt{2rx - xx}}$ pro qua aſſumatur $\mathrm{B}v^3\sqrt{2rx - xx}$. Erit ejus Fluxio $= -\frac{\mathrm{B}v^2x\dot{x}}{\sqrt{2rx - xx}} + \frac{\mathrm{B}rv^2\dot{x}}{\sqrt{2rx - xx}}$, & Terminus tertius Fluentis ex ſecundo proveniens (namque $\mathrm{F} : \frac{r\dot{x}}{\sqrt{2rx - xx}} = v$) erit $\mathrm{C}v^3$ cujus Fluxio eſt $3\mathrm{C}\dot{v}v^2 = \frac{3\mathrm{C}v^2r\dot{x}}{\sqrt{2rx - xx}}$ qui correſpondet $\frac{\mathrm{B}rv^2\dot{x}}{\sqrt{2rx - xx}}$.

Terminus reſtans $\frac{\mathrm{B}rv\dot{x}\sqrt{2rx - xx}}{\sqrt{2rx - xx}} = \mathrm{B}rv\dot{x}$ poſtulat $\mathrm{D}ux$ cujus Fluxionis Terminus primus $= \frac{\mathrm{D}v\dot{x} \times \sqrt{2rx - xx}}{\sqrt{2rx - xx}} = \mathrm{D}v\dot{x}$; & ſecundus Terminus $= \frac{\mathrm{D}rx\dot{x}}{\sqrt{2rx - xx}}$ dat proximum Terminum Fluentis $= \mathrm{E}\sqrt{2rx - xx}$ cujus Terminus primus $- \frac{\mathrm{E}x\dot{x}}{\sqrt{2rx - xx}} = \frac{\mathrm{D}rx\dot{x}}{\sqrt{2rx - xx}}$. Secundus $\frac{\mathrm{E}r\dot{x}}{\sqrt{2rx - xx}}$ dat $\mathrm{F}v$ pro ſexto Termino, &c. Namque Fluxio ejus $\frac{\mathrm{F}r\dot{x}}{\sqrt{2rx - xx}}$ $= \frac{\mathrm{E}r\dot{x}}{\sqrt{2rx - xx}}$; ſic habebis $v^3x + 3v^2 \times \sqrt{2rx - xx} - rv^3 - 3r^2vx$ $- 3r^3 \times \sqrt{2rx - xx} + 3r^3v =$ Fluenti.

CAP.

# CAP. IX.

## *De Æquationibus Fluxionalibus.*

JAM tandem ad Æquationes Fluxionales devenimus, in quibus etiam cæteris omnibus præivit *Neutonus*, uti *Cap.* primo inſinuavimus; pro hujuſmodi Generis Æquationibus duplicem invenit Methodum alteram Particularem, ut ait & Concinniorem, alteram Generaliorem; a poſteriori incipiemus quam *Newtonus* in Epiſtol. poſteriori ait, *Conſiſtere in Aſſumptione Serei pro Quantitate qualibet Incognita ex qua cætera commode derivari poſſint; & in Collatione terminorum Homologorum Æquationis reſultantis ad Eruendos Terminos aſſumptæ Seriei.* Huic in eodem Genere longo poſt temporis intervallo, ſcil. Ann. 1684 ſucceſſit etiam *Leibnitius*, cujus Exemplum hic deſumemus. Sed prius adnotandum eſt in hac Methodo, an in Fluxione propoſita Quantitas exiſtat omnino cognita necne, in eo enim Caſu Series aſſumenda a Quantitate ſeu Termino invariabili incipienda eſt; alias autem ab Indeterminata cum Coefficiente incognita adfecta, & harnm aſſumptionum acceptæ Fluxiones & Æquatæ Fluxionibus datis, poſt debitam factam Comparationem & Subſtitutionem, dabunt Fluentes quæſitas. Exemplum *Neutonianum* quod etiam *Cheynæus* proponit, hocce eſt, ſit Æquatio Fluxiones primas involvens $y^2\dot{x}^2 - x^2\dot{x}y = dd\dot{x}^2 - dx\dot{x}^2$, in qua quæritur $y$ per $x$; poſita $\dot{x} = 1$; & reducta Æquatione, proveniet $y^2 - x^2\dot{y} - dd + dx = 0$. Jam aſſumit $y = a + bx + cx^2 + dx^3 + ex^4 + fx^5$, &c. quare $y^2 = a^2 + 2abx$

$+ \overline{2ac + b^2}x^2 + \overline{2ad + 2bc}\,x^3 + \overline{2be + 2af + c^2}\,x^4$, &c. Et $- x^2\dot{y} = - bx^2 - 2cx^3 - 3ex^4$, &c. quibus ſubſtitutis Loco ipſorum $y^2$ & $- x^2\dot{y}$, Ordinataque Æquatione, fiet

$$o = \left\{ \begin{array}{rlllll} y^2 = & a^2 + 2abx & + 2acx^2 & + 2aex^3 & + 2bex^4, & \&c. \\ & & + b^2 & + 2be & + 2af & \\ & & & & c^2 & \&c. \\ \ddot{x}^2y = & & - b^2 & - 2c & - 3e^2 & \&c. \\ + dx = & + \quad dx & & & & \\ - dd = & - \quad dd. & & & & \end{array} \right\} = o$$

poſitis jam ſingulis terminis Æqualibus nihilo, determinabuntur Coefficientes ſic, $aa - dd = o$, & $a = d$; $2ab + d = o$, $b = - \frac{d}{2a}$, $- \frac{d}{2a} = - \frac{1}{2}$. $2ac + b^2 - b = o$, & $c = - \frac{3}{8d}$, $e = - \frac{9}{16d^2}$, &c. quare ſubſtitutis hiſce Valoribus determinatis loco indeterminatorum in Æquatione aſſumpta, habebimus $y = d - \frac{1}{2}x - \frac{3}{8d}x^2 - \frac{9}{16d^2}x^3$, &c.

Sed antequam ulterius procedimus Legem generalem pro aſſumptionibus in hiſce omnibus a nemine uti ſcio antehac prolatam (ſed ab omnibus ſuppoſitam) exhibebo, pro Fluxionibus ſecundis, tertiis, &c. ad Infinitum. 1. *Obſervetur ea jamjam tradita pro initio Fluxionis* a *dato* vel *Indeterminato, &c. & ſi jamjam non ſint, fiant omnia Fluxionis membra, ejuſdem ordinis ſubſtituendo unitates*; & 2. Tum *fiat numerus punctationum*, h. e. *Numerus ordinis Fluxionis, Intervallum inter Indices indeterminatæ in aſſumptione, & Æquatio ſic aſſumpta ſemper dabit Fluxionem cum data comparabilem*; hoc modo ſit $a^2\ddot{x} + \dot{x}y^2 = o$. Poſita $\dot{y} = 1$, proveniet $a^2\ddot{x} + x = o$. Aſſumatur ergo $x = by + cy^3 + dy^5 + ey^7$, &c. Si eſſet $\dddot{a}x = x = o$. Aſſumenda eſſet $x = by + cy^4 + dy^7 + cy^{10}$, &c. unde eſſet

$$o = \left\{ \begin{array}{l} x = by + cy^4 + dy^7 + cy^{10} \text{ \&c.} \\ \dddot{a}x = 24cay + 210day^4 + 720acy^7 \text{ \&c.} \end{array} \right\} = o$$

&

& aſſumpta $b = 1$. Invenietur $x = y + \frac{y^4}{24a} + \frac{y^7}{210a}$, &c. ſic pro $a\ddddot{x} + x = 0$, ſeu $a\dddot{x} + \dot{y}^5 x$ poſita $\dot{y} = 1$, aſſumenda eſſet $x = by + cy^6 + dy^{11}$, &c. &c. Sit in aliud Exemplum Problema illud *Leibnitianum* ubi proponit inveniendum Sinum Rectum ex dato Arcu & Radio. Æquatio ex ſua Methodo eſt $a^2\dot{y}^2 = a^2\dot{x}^2 + x^2\dot{y}^2$. Fiat $x = by + cy^3 + ey^5 + fy^7$. &c. & erit $\frac{\dot{x}}{\dot{y}} = b + 3cy^2 + 5ey^4 + 7fy^6$, &c. & Valores debitos ſubſtituendo, &c. determinabuntur Valores Coefficientium $b$, $c$, $e$, $f$, &c. Sed idem multo brevius conſequitur, deſcendendo ad Fluxiones ſecundas. Nam poſita $\dot{y} = 1$, habebitur ex Æquatione ſupra poſita $2a^2\dot{x}\ddot{x} + 2x\dot{x}\dot{y}^2 = 0$ ſeu $a^2\ddot{x} + x\dot{y}^2 = 0$. Erit $\frac{\ddot{x}}{\dot{y}^2} = 2 \times 3cy + 4 \times 5ey^3 + 6 \times 7fy^5$, &c. ut patet capiendo Fluxiones ſecundas Valoris $\frac{\dot{x}}{\dot{y}}$ paulo ante habiti; & Æquatio erit

$$0 = \left\{ \begin{array}{l} x = by + cy^3 + dy^5 + ey^7, \text{\&c} \\ \frac{a^2\ddot{x}}{\dot{y}^2} = 2 \times 3a^2cy + 4 \times 5dy^3 + 6 \times 7ey^5, \text{\&c.} \end{array} \right\} = 0$$

& determinatis Coefficientibus habebitur $x = \frac{y}{1} - \frac{y^3}{1 \times 2 \times 3a^2} + \frac{y^5}{1 \times 2 \times 3 \times 4 \times 5a^4} - \frac{y^7}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7a^6}$ &c. h. e. poſito $a =$ Radio, $x =$ Sinui Recto, & $y =$ Arcui, qui in praxi debet eſſe (inquit) notabilitur minor Radio. Proponatur jam Problema illud ad elevandam Seriem quamcunque, ſeu Radicem Multinomiam ad poteſtatem quamlibet Indeterminatam, ſeu quod eodem redit, ad quamvis ejuſdem extrahendam Radicem. Sit *e. g.* Quantitatas Multinomia ſeu Innitinomia $a + bx + cx^2 + dx^3$, &c, $=$ R; & $\overline{a + bx + cx^2 + dx^3, \text{\&c.}}|^n$, $= R^n = y$, erit $b + 2cx + 3dx^2 + 4ex^3$, &c. $= \dot{R}$ & $nlR = ly$, & $\frac{n\dot{R}}{R} = \frac{\dot{y}}{y}$

&

& $ny\dot{R} - R\ddot{y} = 0$, fac

$$y = A + Bx + Cx^2 + Dx^3 + Ex^4, \text{ \&c. erit}$$

$$\dot{y} = B + 2Cx + 3Dx^2 + 4Ex^3, \text{ \&c. eritque}$$

$$= 0\left\{\begin{array}{l} ny\dot{R} = nbA + \left.\begin{array}{r} nbB \\ 2ncA \end{array}\right\}x \left.\begin{array}{r} +\ nbC \\ +\ 2ncB \\ +\ 3ndA \end{array}\right\}x^2 \left.\begin{array}{r} +\ nbD \\ +\ 2ncC \\ +\ 3ndD \\ +\ 4ncA \end{array}\right\}x^3 \text{ \&c.} \\ -R\ddot{y} = -aB \left.\begin{array}{r} -\ bB \\ -\ 2aC \end{array}\right\}x \left.\begin{array}{r} -\ cB \\ -\ 2bC \\ -\ 3aD \end{array}\right\}x^2 \left.\begin{array}{r} -\ dB \\ -\ 2cC \\ -\ 3bD \\ -\ 4aE \end{array}\right\}x^3 \text{ \&c.} \end{array}\right\} = 0$$

Jam pro determinandis Coefficientibus, ſit $A = a^n$. Erit $nbA = aB$. $\frac{nbA}{a} = B = nba^{n-1}$: Et $nbB + 2ncA - bB - 2aC = 0$, &c. Et $C = \frac{n - 1 \times bB + 2ncA}{2a}$, &c. &c. quæ explicatæ coincidunt cum Terminis *Moivræanis* initio hujus Libri expreſſis, ut experienti cuivis facile patebit.

Eadem etiam Methodo peraguntur Tranſlationes Serierum ab una Indefinita Quantitate in aliam, ut & Reverſiones Serierum (abſque Conſideratione Fluxionnm) ſit *v. g.* Exemplum illud *Neutoni*, pag. 33. Analyſ. per Quantitat. Inf. &c. in Epiſtol. ſecunda olim ad *Oldenburg*. Conſcripta $z = ay + by^2 + cy^3 + dy^4 + ey^5$, &c. invenire $y$ in Terminis $z$. Aſſumatur $y = Az + Bz^2 + Cz^3 + Dz^4$, &c. & erit

$$0 = \left\{\begin{array}{lllll} ay = & aAz + aBz^2 & + & aCz^3 & \text{\&c.} \\ by^2 = & & & bA^2z^2 + 2bABz^3 & + \text{\&c.} \\ cy^3 = & & & cA^3z^3 & + \text{\&c.} \end{array}\right\} = 0$$

& poſita $a = 1$, erit $aA = 1$, & $A = \frac{1}{a}$. Rurſus fac $aB + bA^2 = 0$, ſeu $aB = - bA^2$ & erit $B = \frac{bA^2}{a} = -\frac{b}{a^3}$. Et poſita $aC + 2bAB + cA^3 = 0$, erit $aC = - 2bAB - cA^3$ & $C = \frac{2bb - ac}{a^5}$, & ſic porro. Et erit $y = \frac{z}{a} + \frac{b}{a^3}z^2 + \frac{2b^2 - ac}{a^5}z^3$, & ſic porro. Rurſus ſit $z = ay$

+

$+ by^3 + cy^5 + dy^7$, &c. aſſumatur $y = Az + Bz^3 + Cz^5$, & invenietur $y = \frac{z}{a} - \frac{b}{a^4}z^3 + \frac{3b^2 - ac}{a^7}z^5 + \frac{8abc - a^2d - 12b^3}{a^{10}}z^7$, &c. ſic ſi proponeretur Æquatio illa ad Arcum Circuli $z = y + \frac{y^3}{6rr} + \frac{3y^5}{40r^4} + \frac{5y^7}{112r^6} +$ &c. Aſſumenda eſſet $y = Az + Bz^3 + Cz^5 + Dz^7$, &c. & proveniret tandem $y = z - \frac{z^3}{6r^2} + \frac{z^6}{120r^4} - \frac{z^7}{5040r^6}$ &c. & hac Methodo, uti ait *Neutonus*, pag. 33. & poſt eum *Leibnitius*, pag 180. Act. Erud. conſtat Radices affectarum Æquationum poſſe extrahi.

*Vid.* Neut. *Pag.* 34. *Epiſt. prima.*

Sit jam $az + bz^2 + cz^3$, &c. $= gy + hy^2 + iy^3$, &c. invenire $z$ in Terminis $y$. Sit, ſeu aſſumatur $z = Ay + By^2 + Cy^3 + Dy^4 + Ey^5$, &c.

$$\begin{array}{lllllllllll}
az = aAy & +aB & \Big\}y^2 & +aC & & +aD & & +aE & & +aF & \\
bz^2 = \ldots & bA^2 & & +2bAB & & +2bAC & & +2bAD & & +2bEA & \\
 & & & & \Big\}y^3 & bB^2 & & +2bBC & & +2bBD & \\
 & & & & & & & & & bCC & \\
cz^3 = \ldots\ldots & & & cA^3 & & +3A^2B & \Big\}y^4 & +3AB^2c & \Big\}y^5 & +3A^2Dc & \Big\}y^6\ \text{\&c.} \\
 & & & & & & & 3A^2Cc & & +6ABCc & \\
 & & & & & & & & & B^3c & \\
dz^4 = \ldots\ldots\ldots & & & & & dA^4 & & +2dA^3 & & +2A^3Cd & \\
 & & & & & & & & & A^2B^2d & \\
ez^5 = \ldots\ldots\ldots\ldots & & & & & & & eA^5 & & +5eA^4B & \\
fz^6 = \ldots\ldots\ldots\ldots\ldots & & & & & & & & & fA^6 &
\end{array}$$

Jam comparentur Termini Homologi, & habebis, 1°. $aA = g$, ſeu $A = \frac{g}{a}$. 2°. $aB + bA^2 = h$, & $aB = h - bA^2$.

Unde $B = \frac{h - bA^2}{a}$. 3°. $aC + 2bAB + cA^3 = i$ unde $C = \frac{i - 2bAB - cA^3}{a}$. 4°. $aD + 2bAC + bB^2 + 3cA^2B + dA^4 = k$.

Unde $D = \frac{k - 2bAC - bB^2 - 3cA^2B - dA^4}{a}$. Pariter invenietur $E = \frac{l - 2bAD - 2bBC - 3cAB^2 - 3cA^2C - 2dA^3 - eA^5}{a}$, &c. &c. ſubſtituantur hæ Coefficientes determinatæ Loco indeterminatarum in Æquatione aſſumpta & habebitur quæſitum.

Sit Fluxio data Logarithmica $\begin{cases} \dot{y}xlx + \dot{x}xy - \dot{x}lx = 0 \text{ ſeu Poſ. } \dot{x} = 1. \\ \dot{y}xlx + xy - lx = 0 \end{cases}$

Aſſumatur $y = alx + bxlx + cx^2lx + dx^3lx$. &c.

Erit $\dot{y} = ax^{-1} + blx + b + 2cxlx + cx + 3dx^2lx + dx^2$

$\dot{y}xlx = alx + bxl^2x + bxlx + 2cx^2l^2x + cx^2lx + 3dx^3lx$, &c.
hoc eſt $= alx + bxlx + cx^2lx +$ &c. &c.

Et reſtabunt Fluxiones primæ alterius Ordinis, *viz*,

$bxl^2x\dot{x} + 2cx^2l^2x\dot{x}$, &c. pro quibus aſſumenda erit,

$\alpha x^2l^2x + \beta x^3l^2x$, &c. &c. ſic ex aſſumptione prima emerget 2da, &c.

Hæcce Methodus ad altiores Æquationes & plures incognitas Involventes Quantitates facillime provehitur, ſed hæc ſufficiant.

Altera *Neutoni* Methodus conſiſtit *in Extractione Fluentis Quantitatis ex Æquatione ſimul involvente Fluxionem ejus*, neque brevius & clarius, quam ex ipſius verbis, exprimi poſſit, ſcil. hoc proponit.

## PROBLEMA.

*Ex Æquatione Fluxionem Radicis involvente Radicem extrahere.*

### *RESOLUTIO.*

“ TErmini omnes, ex eodem Æquationis latere conſiſtentes, æquen-
“ tur nihilo, & ipſarum $y$ & $\dot{y}$ dignitates (ſi opus ſit) exalten-
“ tur vel deprimantur, ſic ut Indices nec alicubi Negativi ſint, nec ta-
“ men altiores quam ad hunc effectum requiritur; & ſit $kz^{\lambda}$ Terminus
Infi-

“ Infimus eorum qui neque per $y$ neque per ejus Fluxionem $\dot y$, neque per earum dignitatem quamvis multiplicantur. Sit $lz^{\mu} y^{\alpha} \dot y^{\beta}$ Terminus alius quilibet, & omnes ordine Terminos percurrendo collige ex ſingulis ſeorſim Numerum $\frac{\lambda - \mu + \beta}{\alpha + \beta}$, ſic ut tot habeas ejuſmodi Numeros quot ſunt Termini. Horum Numerorum maximus vocetur $v$, & $z^{v}$ erit dignitas primi Termini Seriei. Pro ejus Coefficiente ponatur $a$, & in Æquatione quæ Reſolvenda dicitur ſcribe $az^{v}$ pro $y$, & $vaz^{v-1}$ pro $\dot y$; ac Termini omnes reſultantes in quibus $z$ ejuſdem eſt dignitatis ac in Termino $kz^{\lambda}$, ſub propriis Terminis collecti, ponantur æquales nihilo. Nam hæc Æquatio debite reducta dabit Coefficientem $a$. Sic habes $az^{v}$ Terminum primum Seriei.

## *Operatio Secunda.*

“ Pro reliquis omnibus hujus Seriei Terminis nondum inventis pone $p$, & habebis Æquationem $y = az^{v} + p$, & inde etiam per Methodum directam Æquationem $\dot y = vaz^{v-1} + \dot p$. In Reſolvenda pro $y$ & $\dot y$ ſcribe hos eorum Valores & habebis Reſolvendam novam ubi $p$ præſtat Officium ipſius $y$: Et ex hac Reſolvenda primum extrahes Terminum Seriei $p$ eodem modo atque Terminum primum Seriei totius $y = az^{v} + p$ ex Reſolvenda prima extraxiſti.

## *Operatio Tertia, & ſequentes.*

“ Dein tertiam Reſolvendam eadem ratione invenias atque ſecundam inveniſti, & ex ea Terminum tertium Seriei totius extrahes.

“ Et ſimiliter Reſolvendam quartam invenies, & ex ea quartum Seriei Terminnm, & ſic in Infinitum. Seriei autem ſic inventa erit Radix Æquationis quam extrahere oportuit.

## EXEMPLVM.

“ Ex Æquatione [quam altera Methodo jamjam reſolvimus] $y^{2}\dot z^{2} - z^{2}z\ddot y - dd\dot z\dot z + dz\dot z^{2} = 0$ extrahenda ſit Radix $y$. Pone,

$\dot z = 1$

“ $\dot{z}=1$, & Æquatio evadet $y^2-z^2\dot{y}-dd+dz=0$, quæ eſt reſolvenda. Jam vero Terminus infimus in quo nec $y$ neque $\dot{y}$ reperitur eſt $dd$ qui ipſi $kz^{\lambda}$ Æquatus dat $\lambda=0$; Terminis reliquis $y^2, -z^2\dot{y}$ pone $lz^{\mu}y^{\alpha}\dot{y}^{\beta}$ æqualem ſucceſſive, & inde in primo Caſu habebis $\mu=0$, $\alpha=2$, $\beta=0$, & in ſecundo $\mu=2$, $\alpha=0$, & $\beta=1$. Et hinc $\frac{\lambda-\mu+\beta}{\alpha+\beta}$ fit in primo Caſu 0, in ſecundo $-1$; unde $\nu$ eſt 0 & $az^{\nu}$ & $\nu az^{\nu-1}$ ſunt $a$ & 0; quarum ultimæ duæ $a$ & 0 in Reſovenda pro $y$ & $\dot{y}$ ſcriptæ, producunt $aa-0z^2-dd+dz$; & Termini $aa$ & $-dd$ in quibus Index dignitatis $z$ eſt $\lambda$ ſeu 0, poſiti æquales nihilo dant $a=d$. Unde primus Seriei Terminus $az^{\nu}$ evadit $d$.

### *Operatio Secunda.*

“ Pro Terminis reliquis pone $p$, & habebis Æquationem $y=d+p$, & inde per Methodam directam $\dot{y}=\dot{p}$, qui Valores in Reſolvenda pro $y$ & $\dot{y}$ ſubſtituti dant Reſolvendam novam $2dp+pp-zz\dot{p}+dz=0$ ubi $p$ & $\dot{p}$ vices ſubeunt ipſarum $y$ & $\dot{y}$. Terminus Unicus in quo nec $p$ neque $\dot{p}$ reperitur eſt $dz$, qui cum Termino $kz^{\lambda}$ collatus dat $\lambda=1$. Terminis reliquis $2dp$, $pp$, $-zz\dot{p}$ pone $lz^{\mu}p^{\alpha}\dot{p}^{\beta}$ æqualem ſucceſſive; & inde in primo Caſu habebis $\mu=0$, $\alpha=1$, & $\beta=0$; & in tertio $\mu=2$, $\alpha=0$, & $\beta=1$. Et hinc $\frac{\lambda-\mu+\beta}{\alpha+\beta}$ evadet in primo Caſu 1, in ſecundo $\frac{1}{2}$, in tertio 0. Unde $\nu$ eſt 1, & $az^{\nu}$, & $\nu az^{\nu-1}$ ſunt $az$ & $a$. Termini duo ultimi $az$ & $a$ in Reſolvenda pro $p$ & $\dot{p}$ reſpective ſcripti producunt $2daz+a^2z^2-az^2+dz$. Et Termini $2daz$ & $dz$ in quibus Index dignitatis $z$ eſt $\lambda$ ſeu 1, poſiti Æquales nihilo, dant $a=-\frac{1}{2}$. Unde $az^{\nu}$ Terminus primus Seriei fit $-\frac{1}{2}z$.

*Operatio*

## *Operatio Tertia.*

“ Pro Terminis reliquis nondum inventis pone $q$, & habebis Æquationem
“ $p = -\frac{1}{2} + q$, & inde per Methodum directam $\dot{p} = -\frac{1}{2} + \dot{q}$:
“ qui Valores pro $p$ & $\dot{p}$ in reſolvenda noviſſima ſubſtituti producunt
“ Reſolvendam novam $2d\dot{q} - zq + qq - \frac{3}{4}zz - zz\dot{q} = 0$. Ubi $q$ & $\dot{q}$
“ vices ſupplent ipſorum $y$ & $\dot{y}$. Terminus Unicus in quo neque $q$ neque
“ $\dot{q}$ reperitur eſt $\frac{3}{4}zz$, qui cum $kz^{\lambda}$ collatus dat $\lambda = 2$; Terminis reliquis
“ $2dq$, $-zq$, $+qq$, $-zz\dot{q}$ pone $lz^{\mu} q^{\alpha} \dot{q}^{\beta}$ æqualem ſucceſſive; & inde
“ in primo Caſu habebis $\mu = 0$, $\alpha = 1$, $\beta = 0$; in ſecundo $\mu = 1$,
“ $\alpha = 1$, $\beta = 0$; in tertio $p = 0$, $\alpha = 2$, $\beta = 0$; in quarto $\mu = 2$,
“ $\alpha = 0$, $\beta = 1$; & inde $\frac{\lambda - \mu + \beta}{\alpha + \beta}$ evadit in primo Caſu 2, in ſecundo,
“ tertio, & quarto 1; & hinc $v$ eſt 2, vel $az^{v}$ & $vaz^{v-1}$ ſunt
“ $az^{2}$ & $2az$ qui Valores in Reſolvenda pro $q$ & $\dot{q}$ ſubſtituti dant
“ $2daz^{2} - az^{3} + aaz^{4} + \frac{3}{4}zz - 2az^{3}$; & termini $2dazz + \frac{3}{4}zz$ in
“ quibus Index dignitatis $z$ eſt $\lambda$ ſeu 2, poſiti æquales nihilo, dant
“ $a = -\frac{3}{8d}$. Unde $az^{v}$ Terminus primus Seriei $q$ evadit $-\frac{3zz}{8d}$.

## *Operatio Quarta.*

“ Pro reliquis Terminis nondum inventis pone $r$, & habebis
“ Æquationes $q = -\frac{3zz}{8d} + r$ & $\dot{q} = -\frac{3z}{4d} + \dot{r}$; & inde reſolvendam
“ novam $2d\dot{r} + \frac{9z^{3}}{8d} - zr + \frac{9z^{4}}{64dd} - \frac{zzr}{4d} + rr - zz\dot{r} = 0$, &
“ ex ea per Methodum ſuperiorem habebis $-\frac{9z^{3}}{16dd}$ Terminum primum
“ Seriei $r$, & ſic pergitur ad Infinitum. Eſt igitur Radix extrahenda

T

“ $y =$

“ $y = d + p = d - \frac{1}{2}z + q = d - \frac{1}{2}z - \frac{3zz}{8d} + r = d - \frac{1}{2}z - \frac{3zz}{8d}$
“ $- \frac{9z^3}{16dd} -$ &c. & Operationem continuando licet radicem producere
“ ad Terminos plures.

Hujuſce Methodi Fundamentum patefacit D. *Cheynæus*, pag. 42, 43, 44. ſuæ Methodi Inverſæ, & qui ad Fluxiones ſecundas, tertias, ſuperioreſque porrigatur, *pag*. 44, 45, & 46. oſtendit, ubi terminum infimum ſemper ponit (uti *Neutonus* in præcedentibus) $kz^{\lambda}$, & pro reliquis in unoquoque ſucceſſive Fluxionum gradu hancce Seriem ſcil.

$$Lz^{\mu}y^{\alpha}\dot{y}^{\beta}\ddot{y}^{\gamma}$$

$$Lz^{\mu}y^{\alpha}\dot{y}^{\beta}\ddot{y}^{\gamma}\dddot{y}^{\delta}$$

$Lz^{\mu}y^{\alpha}\dot{y}^{\beta}\ddot{y}^{\gamma}\dddot{y}^{\delta}\ddddot{y}^{\epsilon}$ quibus reſpondent pro maximis Numeris hæcce pariter Series

$$\frac{\lambda - \mu + \beta + 2\gamma}{\alpha + \beta + \gamma} = v = \text{Maximo Numero.}$$

$$\frac{\lambda - \mu + \beta + 2\gamma + 3\delta}{\alpha + \beta + \gamma + \delta} = v$$

$\frac{\lambda - \mu + \beta + 2\gamma + 3\delta + 4\epsilon}{\alpha + \beta + \gamma + \delta + \epsilon} = v$, &c. ad Infinit. in quibus $v$ indicabit poteſtatem ipſius $z$, ad quam elevata æquabitur ipſi $y$ proxime

Ponendum, uti inquit, pro $\ddddot{y}$ . . $v - 3 \times v - 2 \times v - 1 \times vz^{v-4}$
$\dddot{y}$ . . . . . $v - 2 \times v - 1 \times vz^{v-3}$
$\ddot{y}$ . . . . . . . . . . $v - 1 \times vz^{v-2}$
$\dot{y}$ . . . . . . . . . . . . . $vz^{v-1}$
$y$ . . . . . . . . . . . . . $z^{v}$

& ſubſequenti pagina eandem accommodat ad æquationem tres indeterminatas $y$, $z$, $v$, cum ipſarum Fluxionibus involventem, &c. hujuſce

“ Methodi *Neutonianæ* ſit in aliud Exemplum, *Cheynæanum* illud, *p.* 106.
“ Sit Æquatio ad Aream Hyperbolæ $A = y - \frac{y^2}{2} + \frac{y^3}{3} + \frac{y^4}{4}$, &c. ſeu
“ $-A + y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4}$, &c. $= 0$, jam infimus Terminus in quo
“ nec $y$, neque ipſius poteſtas quævis reperitur eſt A, qui æquatus ipſi
“ $kA^\lambda$ dat $\lambda = 1$, Terminis reliquis $y, -\frac{y^2}{2}, +\frac{y^3}{3}, -\frac{y^4}{4}$, &c. ponendo
“ $LA^\mu y^\alpha$ æqualem ſucceſſive, habebis in primo caſu $\mu = 0$, $\alpha = 1$, in
“ ſecundo $\mu = 0$, $\alpha = 1$, in tertio $\mu = 3$, $\alpha = 3$, & ſic deinceps. Hinc
“ $\frac{\lambda - \mu}{\alpha}$ in primo Caſu $= 1$, in ſecundo $= \frac{1}{2}$, in tertio $= \frac{1}{3}$, &c.
“ quare $\nu = 1$, $\alpha A^\nu = \alpha A$ qui ſubſtitutus in Reſolvenda Loco $y$, dat
“ $-A + \alpha A + \frac{\alpha^2 A^2}{2}$, &c. $= 0$. Adeoque Termini in quibus A eſt
“ ejuſdem dignitatis, poſiti æquales nihilo, dant $\alpha = 1$. Unde primus
“ Seriei Terminus eſt A. Pro reliquis pone $p$, erit $z = A + p$, qui
“ Valor ſubſtitutus in Reſolvenda loco $y$, dat $-A + A + p -$
“ $\frac{A^2 + 2Ap + p^2}{2} + \frac{A^3 + 3A^2p + 3Ap^2 + p^3}{3}$, &c. $= 0$. Terminus
“ jam infimus in quo nec $p$, nec ipſius Poteſtas quævis reperitur, eſt
“ $-\frac{A^2}{2}$ qui æquatus ipſi $kA^\lambda$ dat $\lambda = 2$, Terminis reliquis $p, -Ap$,
“ $-\frac{p}{2}, + \frac{A^3}{3}$, &c. Ponendo $LA^\mu p^\alpha$ æqualem ſucceſſive habebis in
“ primo Caſu $\mu = 0$, $\alpha = 1$; in ſecundo $\mu = 1$, $\alpha = 1$; in tertio
“ $\mu = 0$, $\alpha = 2$; in quarto $\mu = 3$, $\alpha = 0$, &c. Unde $\frac{\lambda - \mu}{\alpha}$ in primo
“ Caſu eſt 2; in ſecundo $\frac{1}{2}$; in tertio 1, & ſic in cæteris. Quare
“ $\nu = 2$, $\alpha A^\nu = \alpha A^2$, loco $p$ ſubſtituatur $\alpha A^2$, & Termini in quibus A
“ eſt ejuſdem dignitatis poſiti æquales nihilo dant $\alpha A^2 - \frac{A^2}{2} = 0$,

“ adeoque $\alpha = \frac{1}{2}$. Pro Terminis reliquis nondum inventis pone $q$, &
“ habebis Æquationem $p = \frac{1}{9}A^2 + q$, qui ſubſtitutus in Reſolvenda
“ producit Reſolvendam novam, quæ tractata ut ſuperiores, exhi-
“ bebit tertium Seriei Terminum $\frac{A^3}{6}$, & ad hunc modum proſequendo
“ præcepta Methodi ſupra traditæ habebis tandem $y = A + \frac{A^2}{2} + \frac{A^3}{3}$
“ $\frac{A^4}{4}$, &c.

Duabus hiſce Methodis *Neutonianis* hoc modo expoſitis, reſtat adhuc tertia *Johannis Bernoullii* quam hic quoque recenſere libet. Eandem invenire licet in *Act. Eruditor. Lipſiæ*, Ann. 1694, & ejuſdem meminit D. *Cheynæus* Meth. Inverſ. *pag.* 50. hiſce verbis *Fluxionis datæ Fluens, inquit, per Seriem infinitam generalem quidem, terminis tamen plerumque maxime implicitis, obtinetur.* Seriem ipſam generalem ibidem adducit & inveſtigat, ſed quoniam magis particularem ejuſdem inveſtigationem in *Rudim.* affert, poſteriorem hic ſolam coram ſiſtemus, ſcil. *Sit* $\dot{z}y$ *expreſſio generalis pro Fluxionibus primis quibuſcunque (ubi* $y$ *eſt Quantitas quomodocunque ex indeterminatis & conſtantibus compoſita) ſit jam iſtius Fluens* $Az^m y^n$ *erit* $mAz^{m-1}\dot{z}y^n + nAz^m y^{n-1}\dot{y}$ *ipſius Fluxio: Jam poſterior terminus Fluxionis aſſumptæ Æquationis, cum Fluxionis propoſita comparari nequit, quia in eo nullus exiſtit* $\dot{z}$ *(& ut comparatio procedat, requiritur ſemper ut omnes indeterminatæ cum earum Fluxionibus reperiantur in Fluxione aſſumptæ Æquationis, quæ reperiuntur in propoſita; ſin plures adſint nihil periculi inde Metuendum, quia poſitis iis omnibus æqualibus Unitati evaneſcunt.) Adeoque eſt prior Terminus Fluxionis aſſumptæ Æquationis comparandus cum Fluxione propoſita*, ſcil. $mAz^{m-1}\dot{z}y^n = \dot{z}y$, unde $m - 1 = 0$, $mA = 1$, $n = 1$. *Et Fluentis Terminus primus eſt* $zy$, *cujus Fluxio* $\dot{z}y + z\dot{y}$, *ſed cum jam reſtiterit* $\dot{y}z$ *cui nullus eſt repertus Fluentis Terminus comparabilis, eum pono* $Bz^m\dot{y}^n$ *(non* $y^n$ *quia ſic coincideret inveniendus cum Termino ante reperto) igitur* $mB\dot{z}z^{m-1}\dot{y}^n + nBz^m\dot{y}^{n-1}\ddot{y}$ *eſt ipſius Fluxio, quia autem ſolus primus Terminus comparari poterit cum Fluxionis reliquo Termino, ex aſſumpto primo Fluentis Termino nato, (alias* $\ddot{y}$ *evadet = Uinitati & ſic Comparationes ulterius non procedent, ob defectum Terminorum homologorum, uti experienti*

*enti patebit) erit* $m - 1 = 1$, & $m = 2$, $n = 1$; $\dot{z} = 1 =$ *Conſtanti (ideoque ejus Fluxiones ulteriores nullæ) & hiſce ſubſtitutis loco illorum, fit* $2Bz\dot{y} + z\dot{y} = 0$, & $B = -\frac{1}{2}$. *Et ſic ſecundus Fluentis Terminus evadit* $-\frac{z^2\dot{y}}{2}$, *ſed quia reſtat hic etiam Terminus* $\frac{z^2\ddot{y}}{2}$ *cui nullus eſt repertus Fluentis terminus comparandus, pono eum* $Cz^m\ddot{y}^n$ & ſic $mCz^{m-1}\dot{z}\ddot{y}^n + nCz^m\ddot{y}^{n-1}\dddot{y}$, *erit ejus Fluxio. Et quia comparando ſecundum terminum hujus cum Fluentis reliquo, evadit* $\ddot{y} = 1$, *& ſic comparationes impoſterum ſiſterentur, ideo primum cum reliquo comparo, & invenio* $m - 1 = 2$; *adeoque* $m = 3$, $\dot{z} = 1$, $n = 1$, *& hiſce ſubſtitutis loco illorum fit* $3Cz^2\ddot{y} - \frac{1}{2}z^2\ddot{y} = 0$; *& Coefficientem determinando, fit* $C = \frac{1}{1 \times 2 \times 3}$, *& ſic deinceps. Adeoque Fluens ipſius* $\dot{z}y$ *eſt*

$$zy - \frac{z^2\dot{y}}{1 \times 2} + \frac{z^3\ddot{y}}{1 \times 2 \times 3} - \frac{z^4\dddot{y}}{1 \times 2 \times 3 \times 4},$$

*&c. & quia ubique* $\dot{z} = 1$ *(ut Termini evadant homologi)*

$$zy - \frac{z^2\dot{y}}{1 \times 2\dot{z}} + \frac{z^3\ddot{y}}{1 \times 2 \times 3\dot{z}^2} - \frac{z^4\dddot{y}}{1 \times 2 \times 3 \times 4\dot{z}^3}, \text{ \&c.}$$

*& hinc tandem tuto aſſumere licet (ex perſpecto Fluxionis genio)* $azy + \frac{bz^2\dot{y}}{\dot{z}} + \frac{cz^2\ddot{y}}{\dot{z}^2}$, *&c. quouſque libuerit. Et* Hæc Series *(pergit* Bernoulli) *vicem Canonis præſtabit pro Fluxionibus quibuſcunque in Seriem conjiciendis, in caſibus enim particularibus evaneſcunt* $\dot{y}$, $\ddot{y}$, $\dddot{y}$, $\ddddot{y}$, & $\dot{z}$, $\dot{z}^2$, $\dot{z}^3$, *&c. quia in Caſu dato, dantur harum relationes in terminis* Algebraicis, *& conſequenter Series in Caſibus particularibus conſtabit Terminis pure Algebraicis.*

*Exemplum* V. C. *hoc eſt. Sit* $\dot{y} = \frac{a\dot{x}}{r}$ *(ubi* $r = x + a$) *factis in Canone* $z = x$, $\dot{z} = \dot{x}$, $y = \frac{a}{r}$, $\dot{y} = ($*ob* $\dot{r} = \dot{x}) - \frac{a\dot{x}}{rr}$, $\ddot{y} = \frac{1 \times 2a\dot{x}^2}{r^3}$,

$\dddot{y} = \frac{1\times 2\times a\dot{x}^3}{r^4}$, *&c. fietque* $y = \frac{ax}{r} + \frac{ax^2}{2r^2} + \frac{ax^3}{3r^3} + \frac{ax^4}{4r^4}$, &c.

Sit $z\dot{y}v$ Expreſſio generalis pro Tranſcendentibus vel Exponentialibus primi gradus (ubi $y$ eſt Quantitas Algebraica ex Indeterminatis & conſtantibus quomodocunque compoſita, $v$ vero Quantitas Geometrice irrationalis quæcunque) eodem modo aſſumatur $Az^my^nv^p$ pro primo Termino, ex ejus Fluxione provenient

$mA\dot{z}z^{m-1}y^nv^p \left|\underline{+ nAz^m\dot{y}y^{n-1}v^p + pAz^my^n\dot{v}v^{p-1}}\right.$ pro primis Terminis reſtantibus. Fac $z\dot{y}v = mA\dot{z}z^{m-1}y^nv^p$, & habebis $m-1=0$, ſeu $m=1$, $n=1$, $p=1$, $mA=1$, $A=\frac{1}{m}=1$, & $Az^my^nv^p = zyv$, jam autem proximo loco reſtat $nAz^m\dot{y}^{n-1}yv^p = nAz^m\dot{y}v^p$ (ob $n-1=0$.) $= z\dot{y}v$.

Aſſumatur $Bz^m\dot{y}^nv^p$ & habebis

$mB\dot{z}z^{m-1}\dot{y}^nv^p \left|\underline{+ nBz^m\dot{y}^{n-1}\ddot{y}v^p \left|\underline{+ pBz^m\dot{y}^n\dot{v}v^{p-1}}\right.}\right.$, & $mB\dot{z}z^{m-1}\dot{y}^nv^p = z\dot{y}v$. Unde $m-1=1$, $m=2$, $mB=1$, $B=\frac{1}{2}$, $n=1$, $p=1$, & $\dot{z}=1$. Unde $Bz^m\dot{y}^nv^p = \frac{z^2\dot{y}v}{1\times 2}$ jam proxime reſtat $nBz^m\dot{y}^{n-1}\ddot{y}v^p = nBz^m\ddot{y}v^p = \frac{z^2\ddot{y}v.}{1\times 2}$ Pro qua aſſumatur $Cz^m\ddot{y}^nv^p$, & habebis $mCz^{m-1}\ddot{y}^nv^p$ $\left|\underline{+ nCz^m\ddot{y}^{n-1}\dddot{y}v^p \left|\underline{+ pCz^my^n\dot{v}v^{p-1}}\right.}\right.$ & $mCz^{m-1}\ddot{y}^nv^p = \frac{z^2\ddot{y}v}{2}$, & $m-1=2$, $m=3$, $mC=\frac{1}{2}$, $C=\frac{1}{2\times 3}$, $n=1=p$, Unde $Cz^m\ddot{y}^nv^p = \frac{z^2\ddot{y}v}{1\times 2\times 3}$. Jam autem reſtat rurſus, $nCz^m\ddot{y}^{n-1}\dddot{y}v^p = nCz^m\dddot{y}v^p = \frac{z^3\dddot{y}v}{1\times 2\times 3}$. Aſſumatur $Dz^m\dddot{y}^nv^p$, & habebis

$mDz^{m-1}\dddot{y}^nv^p \pm \left|\underline{nDz^m\dddot{y}^{n-1}\ddddot{y}v^p \left|\underline{+pDz^my^n\dot{v}v^{p-1}}\right.}\right.$, & erit $mDz^{m-1}\dddot{y}^nv^p =$ $z^3\dddot{y}v$

$\frac{z^3\overset{\therefore}{y}v}{6}$. Unde $m-1=3$, & $m=4$, $n=1=p$, $mD=\frac{1}{6}$ & $D=\frac{1}{4\times 6}$, &c. Eodem modo prodibit $Ez^m\overset{::}{y}^nv^p=\frac{z^5\overset{::}{y}v}{1\times 2\times 3\times 4\times 5}$ & $Fz^m\overset{\therefore}{\ddot{y}}^nv^p=\frac{z^6\overset{\therefore}{\ddot{y}}v}{1\times 2\times 3\times 4\times 5\times 6}$, &c. ad Inf. & reſtabit Series Fluxionum hæcce, *viz.* $pAz^my^n\dot{v}v^{p-1}+pBz^my^n\dot{v}v^{p-1}+pCz^my^n\dot{v}v^{p-1}+pDz^my^n\dot{v}v^{p-1}$, &c. ad Inf. pro quibus aſſumendi ſerunt $Ez^my^nv^p+Fz^my^nv^p+Gz^my^nv^p$, &c. ad Inf. Adeo ut hæc quoque Fluens erit Infinities Infinita ; & de hujuſmodi Seriebus, ſive Logarithmicis, ſive Irration. *Craig.* ſive Fluxionalibus *Bernoull.* Irration. juſte ſatis dici poſſit, quod in alio in Caſu profert *Juvenalis*

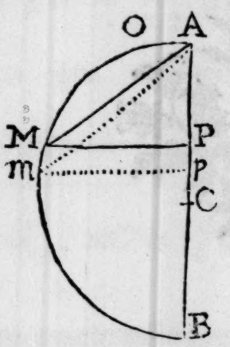
*Volveris a primis quæ proxima* ———

CAP.

# CAP. X.

## *De Methodo inveniendi Fluentes per Figuras Geometricas,* &c.

UTI Fluentes Fluxionum primarum debite tractatæ exhibent inter cætera Curvarum Areas, ſic etiam viciſſim Areæ Curvarum poſſint exhibere Fluxionum primarum Fluentes; & hinc exurgit Methodus inveniendi Fluentes per Figuras, vel Geometricas vel Mechanicas adeo ut invenire Fluentes Fluxionum primarum æquipollet Qurdraturæ Curvarum & reciproce, ſeu invenire Quadraturas æquipollet Fluentibus Fluxionum primarum; Sit *e.g.* Fluxio data $\dot{x}\sqrt{2rxx - xx}$.



Centro C, Radio CB $= r$ deſcribatur Semicirculus AMB, ſit AP $= x$, erit PB $= 2r - x$ & MP $= \sqrt{2rx - xx}$, & P$p = \dot{x}$, unde Fluxio Areæ ſeu Parallelogrammum M$p = \dot{x}\sqrt{2rx - xx}$, & conſequenter ſumma omnium $\dot{x}\sqrt{2rx - xx}$, ſeu Fluens = Semiſegmento AMB.

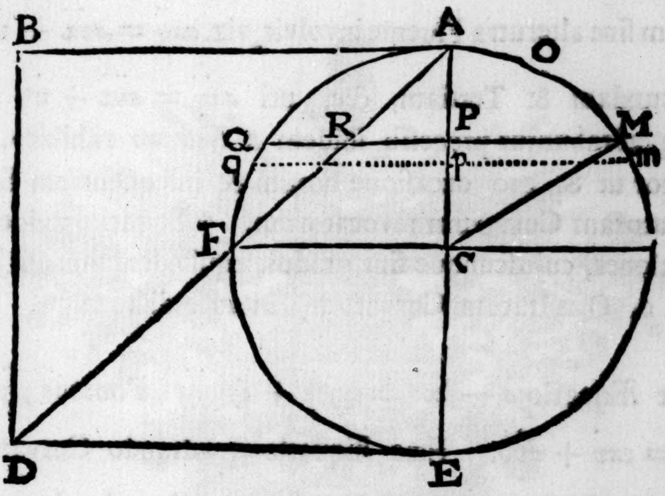
Invenienda eſt jam Fluens $\frac{rx\dot{x}}{2\sqrt{rx - xx}}$. Ducantur AM, A*m* ad diſtantiam una ab altera infinite parvam, MP, *mp* Diametro AB inſiſtentes ad Angulos rectos, & MR normalis ad A*m*, tum (per Circuli proprietatem) AM $= 2rx$, & R*m* ejuſdem Fluxio $= \frac{r\dot{x}}{\sqrt{2rx}}$.

Quoniam autem Triangula APM, MR*m* ſimilia ſunt (Angulis ſcil. AMP & M*m*R æqualibus Circuli Angulis inſiſtentibus) habebis

$$PM\ (\sqrt{2rx-xx}) : AP\ (x) :: Rm\ \left(\frac{r\dot{x}}{\sqrt{2rx}}\right) : MR = \frac{rx\dot{x}}{\sqrt{2rx}+\sqrt{2rx-xx}}$$

ac proinde ſector infinite parvus $MAR = \frac{1}{2}AR \times MR = \frac{rx\dot{x}}{2\sqrt{2rx-xx}} =$ Fluxioni datæ, unde patet Segmentum AOMA Fluens eſſe Fluxionis datæ.

Requiratur invenire Fluentem $x\dot{x} \times 2\sqrt{2rx-xx}$



Centro C, Radio CA $= r$ deſcribatur Circulus AFEM & ſit AP $= x$, P$p = \dot{x}$, PE $= 2r - x$ Circumferentia AFEM $= c$, & ſumma omnium $\dot{x}x \times 2\sqrt{2rx-xx} = \frac{crr}{2}$.

1°. Sit jam inter Exempla *Neutoniana* Primum, *viz.* Æquatio quæ Fluentem & ejus Fluxionem primam involvit ſine altera Fluente,

$$\text{viz. } aa\dot{v} = av + vv$$

& complendo dimenſiones $av\dot{v} = av\dot{z} + vv\dot{z}$

Tum $\frac{aa\dot{v}}{av + vv} = \dot{z}$. Ponatur jam $\dot{v} = 1$.

Erit $\frac{aa}{av + vv} = \dot{z}$

Et F : $\frac{aa}{av + vv} = z$ ſeu quadrando Curvam cujus Ordinata eſt $\frac{aa}{av + vv}$ & Abſciſſa $v$ habebitur Fluens $z$, ſeu Curvæ Area.

2°. Sic pro Æquatione quæ duas ejuſdem Fluentis Fluxiones primam & ſecundam ſine alterutra Fluente involvit, viz. $aa\ddot{v} = a\dot{v}\dot{z} + \dot{v}\dot{v}$ vel

3°. Secundam & Tertiam, &c. uti $aa\ddot{v} = a\dot{v}\ddot{z} + \dot{v}^3$, & ſic porro ad Inf. quadrabuntur proceſſu ibidem a *Neutono* exhibito, quem conſulat Lector ut & pro occaſione horum & inſequentium Exemplorum ad Quadraturam Curvarum revocatorum. * Et ratio quidem eſt, quia tales Fluxiones, cujuſcunque ſint gradus, æquivalent tantum Fluxionibus primis, h. e. Quadraturæ Curvarum, ut ante dictum.

4°. Sit Æquatio $a - bx^m = cxy^n\dot{y} + dy^{2n}\dot{y}\dot{y}$. Ponatur $y^n\dot{y} = \dot{v}$. Erit $a - bx^m = cx\dot{v} + d\dot{v}\dot{v}$. Hæc Æquatio quadrando Curvam cujus Abſciſſa eſt $x$ & Ordinata $\dot{v}$, dat Aream $v$, & Æquatio altera $\dot{v} = y^n\dot{y}$ regrediendo ad Fluentes dat $v = \frac{1}{n+1}y^{n+1}$, unde habetur Fluens $y$.

5°. Sit $\overline{ax^m + bx^n}|^p = rex^{r-1}y^s + sex^r\dot{y}y^{s-1} - f\dot{y}y^t$ ſit $\dot{x} = 1$. Et pars poſterior $rex^{r-1}y^s + sex^r\dot{y}y^{s-1} - f\dot{y}y^t$ regrediendo ad Fluentes, fit $ex^ry^s - \frac{f}{t+1}y^{t+1}$ quæ proinde eſt ut Area Curvæ cujus Abſciſſa eſt $x$,

---

* *Vid. etiam* Schem. Neutonianum.

&

& Ordinata $\overline{ax^m + bx^n}|^p$, & inde datur Fluens $y$.

6°. Sit $\dot{x} \times \overline{ax^m + bx^n}|^p = \frac{d\dot{y}y^{n-1}}{\sqrt{ef + z^n}}$ & Fluens cujus Fluxio eſt $\dot{x} \times \overline{an^m + bx^n}|^p$ erit ut Area Curvæ cujus Abſciſſa eſt $x$ & Ordinata $\overline{ax^n + bx^n}|^p$ & Fluens cujus Fluxio eſt $\frac{d\dot{y}y^{n-1}}{\sqrt{e + fy^n}}$ erit ut Area Curvæ cujus Abſciſſa eſt $y$ & Ordinata $\frac{dy^{n-1}}{\sqrt{c + fy^n}}$.

* * *

CAP.

# CAP. XI.

## *De Comparatione Curvarum, &c.*

PRO Comparatione Curvarum Seriem Propoſitionum ad Infinitum pergentium exhibet *Neutonus* in Prop. 7 & 8 fundatam Propoſitio autem hujuſmodi eſt.

“ Poſitis $e + fz^{n} + gz^{2n} + hz^{3n}$, &c. ad Inf. $= R$ in Curvæ alicujus “ Ordinata $z^{\theta \pm \sigma n} R^{\lambda \pm \tau}$ maneant Quantitates datæ $\theta$, $n$, $\lambda$, $e$, $f$, $g$, &c. & “ pro $\sigma$ & $\tau$ ſcribantur ſucceſſive numeri quicunque Integri : Et ſi detur “ Area unius ex Curvis quæ per Ordinatas innumeras ſic produntes deſignantur, ſi Ordinatæ ſunt duorum Nominum in Vinculo Radicis, vel ſi dentur Areæ duarum ex Curvis ſi Ordinatæ ſunt trium “ Nominum in Vinculo Radicis, *&c.* dabuntur Areæ Curvarum “ omnium.

“ Sit *e. g.* Curvæ Unius Ordinata $pz^{\theta-1} R^{\lambda-1}$ & Area $pA$ exiſtente “ R quantitate duorum nominum $e + fz^{n}$. Et cum per Prop. 3. ſit $z^{\theta} R^{\lambda}$ Area Curvæ cujus Ordinata $\left.\theta e + \begin{matrix}\theta \\ \lambda n\end{matrix} f\right\} z^{\theta-1} R^{\lambda-1}$ ſubduc Ordinatam & Aream priorem de Ordinata & Area poſteriori & manebit

$$\begin{array}{ll} \theta e + \overline{\theta + \lambda n}\, fz^{n} & \quad z^{\theta} R^{\lambda} \\ -p & \quad -pA \\ \hline \theta e - p + \overline{\theta + \lambda n}\, fz^{n} \times z^{\theta-1} R^{\lambda-1} & \quad z^{\theta} R^{\lambda} - pA = \end{array}$$

Ordinatæ & Areæ Novæ Curvæ. Fac jam $\theta - p = 0$, & Ordinata evadet $\overline{\theta + \lambda n} fz^{n} \times R^{\lambda-1}$, & Area $z^{\theta} R^{\lambda} - \theta eA$; divide utramque per $\theta f + \lambda nf$ & habebis $z^{\theta+n-1}$ pro Ordinata, & $\frac{z^{\theta} R^{\lambda} - eA}{\theta f + \lambda nf} = B =$ Areæ; & aſſumpta utcunque $q$ erit $qB$ Area Curvæ $= \frac{qz^{\theta} R^{\lambda} - q\theta eA}{\theta f + \lambda nf}$ congruentis Ordinatæ $qz^{\theta+n-1} R^{\lambda-1}$.

2°. Sumatur jam Ordinata $\left.\begin{matrix}\theta f\\ \lambda nf\\ -q\end{matrix}\right\} z^n + \left.\begin{matrix}\theta g\\ 2\lambda ng\end{matrix}\right\} z^{2n} \times z^{\theta-1} R^{\lambda-1}$, &

$\theta f + \lambda nf - q = 0$; reſtat ergo $\overline{\theta + 2\lambda n}\, g z^{2n} \times z^{\theta-1} R^{\lambda-1}$ = Novæ Ordinatæ & $z^\theta R^\lambda - \theta f B - \lambda nf B$ pro nova Area. Dividatur pars utraque per $\theta g + 2\lambda ng$, & habebis $z^{2n} \times z^{\theta-1} R^{\lambda-1}$. . . . . $\dfrac{z^\theta R^\lambda - \theta f B - \lambda nf B}{\theta g + 2n\lambda g}$

Et aſſumpta utcunque $r$ erit $rz^{\theta+2n-1} R^{\lambda-1}$. : . $\dfrac{rz^\theta R^\lambda - r\theta f B - r\lambda nf B}{\theta g + 2\lambda ng}$ = Novæ Ordinatæ & Areæ correſpondenti $= rC$.

3°. Sumatur jam $\left.\begin{matrix}\theta g\\ 2\lambda ng\\ -r\end{matrix}\right\} z^{2n} + \left.\begin{matrix}\theta b\\ 3\lambda nb\end{matrix}\right\} z^{3n} \times z^{\theta-1} R^{\lambda-1}$ . . . . . $z^\theta R^\lambda$.

Erit $\theta g + 2\lambda ng - r = 0$, ſeu $\theta g + 2\lambda ng = r$, & invenietur $\theta \overline{+ 3\lambda nb}\, z^{3n} \times z^{\theta-1} R^{\lambda-1}$ = alteri Novæ Ordinatæ & $z^\theta R^\lambda - rC$ = Areæ correſpondenti, hoc eſt (ob $r = \theta g + 2\lambda ng$) $z^\theta R^\lambda - \theta g C - 2\lambda ng C$. Dividatur jam pars utraque per $\theta b + 3\lambda nb$, & habebis $z^{3n} \times z^{\theta-1} R^{\lambda-1}$ & $\dfrac{z^\theta R^\lambda - \theta g C - 2\lambda ng C}{\theta b + 3\lambda nb} = D$, & aſſumpta utcunque $s$ erit Ordinata $sz^{\theta+3n-1} R^{\lambda-1}$, & Area $\dfrac{sz^\theta R^\lambda - s\theta g C - 2s\lambda ng C}{\theta b + 3\lambda nb} = sD$, & ſic porro.

Adeo ut Series Ordinatarum & Arearum ſic ſe habebit

| Ord. | Areæ. |
|---|---|
| $pz^{\theta-1} \quad R^{\lambda-1}$ . . . . . . . . . : | $pA$ |
| $qz^{\theta+n-1} R^{\lambda-1}$ . . . . . . | $z^\theta R^\lambda - pA = \dfrac{qz^\theta R^\lambda - q\theta e A}{\theta f + \lambda nf}$ |
| $rz^{\theta+2n-1} R^{\lambda-1}$ . . . . . . . | $z^\theta R^\lambda - qB = \dfrac{rz^\theta R^\lambda - \theta r f B - r\lambda nf B}{\theta g + 2\lambda ng}$ |
| $sz^{\theta+3n-1} R^{\lambda-1}$ . . . . . . . | $z^\theta R^\lambda - rC = \dfrac{sz^\theta R^\lambda - s\vartheta g C - 2s\theta ng C}{\theta b + 3\lambda nb}$, &c. |

ad Inf. in genere Binomio.

Sit jam $R = e + fz^n + gz^{2n}$. Pro Curvis Trinomiis quæ ope duarum Curvarum datarum quadrantur; ex datis ſcil. Ordinatis $pz^{\theta-1}R^{\lambda-1}$ & $qz^{\theta+n-1}R^{\lambda-1}$ & Areis $pA$ & $qB$.

$$\begin{matrix} & & \text{Ord.} & & \text{Ar.}\\ \text{Aſſumatur} & \theta e + \left.\begin{matrix}\theta f\\ \lambda nf\end{matrix}\right\} z^n & \left.\begin{matrix}+\theta g\\ 2\lambda ng\end{matrix}\right\} z^{2n} \times z^{\theta-1} R^{\lambda-1} & .. & z^\theta R^\lambda\\ \text{Subducantur} & p\ ..\ q\ . & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . & .. & pA\ ..\ qB\end{matrix}$$

$$\overline{\left.\begin{matrix}\theta e +\\ -p\\ \phantom{-q}\end{matrix}\ \begin{matrix}\theta f\\ \lambda nf\\ -q\end{matrix}\right\} z^n \left.\begin{matrix}+\theta g\\ 2\lambda ng\end{matrix}\right\}} z^{2n} \times z^{\theta-1} R^{\lambda-1}\ ..\ \overline{z^\theta R^\lambda - pA - qB} =$$

Novæ Ordinatæ & Areæ. Pone jam $\theta e - p = 0$, & $\theta f + \lambda nf - q = 0$, & Ordinata evadet $\overline{\theta g + 2\lambda ng}\, z^{\theta+2n-1} R^{\lambda-1}$, & Area $z^\theta R^\lambda - \theta eA - \theta fB - \lambda nfB$. Dividatur pars utraque per $\theta g + 2\lambda ng$, & habebis

$$z^{\theta+2n-1}R^{\lambda-1}\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ \frac{z^\theta R^\lambda - \theta eA - \theta fB - \lambda nfB}{\theta g + 2\lambda ng},$$ & aſſumpta utcunque $r$, erit $rz^{\theta+2n-1}R^{\lambda-1} =$ Ordinatæ, &

$$\frac{rz^\theta R^\lambda - \theta reA - \theta rfB - \lambda nfB}{\theta g + 2\lambda ng} = rC = \text{Areæ.}$$

Sunto jam duarum Curvarum Ordinatæ $qz^{\theta+n-1}R^{\lambda-1} \times rz^{\theta+2n-1}\overline{R}^{\lambda-1}$ & Areæ $qB$ & $rC$

$$\text{Aſſumatur} \left.\begin{matrix}\theta e +\\ -p\\ \phantom{-q}\end{matrix}\ \begin{matrix}\theta f\\ \lambda nf\\ -q\end{matrix}\right\} z^n + \left.\begin{matrix}\theta g\\ 2\lambda ng\\ -r\end{matrix}\right\} z^{2n} \left.\begin{matrix}\theta h\\ 3\lambda nh\end{matrix}\right\} z^{3n} \times z^{\theta-1} R^{\lambda-1},\ \text{\& Area } z^\theta R^\lambda.$$

Ponantur $\theta e - p = 0$. $\theta f + \lambda nf - q = 0$, & $\theta g + 2\lambda ng - r = 0$. Et erit $\overline{\theta h + 3\lambda nh}\, z^{3n} \times z^{\theta-1} R^{\lambda-1} =$ Ordinatæ novæ, & $z^\theta R^\lambda - qB - rC =$ ejuſdem Areæ, hoc eſt $z^\theta R^\lambda - \theta fB - \lambda nfB - \theta gC - 2\lambda ngC =$ Areæ; diviſaque utraque parte per $\theta h + 3\lambda nh$, habebis $z^{\theta+3n-1}R^{\lambda-1}$ & $\dfrac{z^\theta R^\lambda - \theta fB - \lambda nfB - \theta gC - 2\lambda ngC}{\theta h + 3\lambda nh}$, & aſſumpta utcunque $s$ erit

$$\frac{sz^\theta R^\lambda - \theta sfB - \lambda nsfB - \theta gsC - 2\lambda ngsC}{\theta h + 3\lambda nh} = \text{Areæ Curvæ cujus Ordinata eſt}$$ $sz^{\theta+3n-1}R^{\lambda-1}$.

Sunto jam duarum Curvarum Ordinatæ $rz^{\theta+2n-1}R^{\lambda-1}$ & $sz^{\theta+3n-1}R^{\lambda-1}$, & earundem Areæ $r$C & $s$D.

Aſſumatur $\left.\begin{array}{l}\theta e+\theta f\\ -p\quad \lambda nf\\ \quad\; -q\end{array}\right\} z^{n} + \left.\begin{array}{l}\theta g\\ 2\lambda ng\\ -r\end{array}\right\} z^{2n} + \left.\begin{array}{l}\theta h\\ 3\lambda nh\\ -s\end{array}\right\} z^{3n} + \left.\begin{array}{l}\theta k\\ 4\lambda nk\end{array}\right\} z^{4n} \times z^{\theta-1} R^{\lambda-1}$,

factis jam $\theta e - p = 0$. $\theta f + \lambda nf - q = 0$. $\theta g + 2\lambda ng - r = 0$. $\theta h + 3\lambda nh - s = 0$. Erit $\overline{\theta + 4\lambda n} \times kz^{4n} \times z^{\theta-1}R^{\lambda-1} =$ Ordinatæ & $z^{\theta}R^{\lambda} - r\text{C} - s\text{D} =$ Areæ. Hoc eſt $z^{\theta}R^{\lambda} - \theta g\text{C} - 2\lambda ng\text{C} - \theta h\text{D} - 3\lambda nh\text{D} =$ Areæ. Dividatur pars utraque per $\theta k + 4\lambda nk$; & habebis $z^{\theta+4n-1}R^{\lambda-1}$ & $\frac{z^{\theta}R^{\lambda} - \theta g\text{C} - 2\lambda ng\text{C} - \theta h\text{D} - 3\lambda nh\text{D}}{\theta k + 4\lambda nk}$ & aſſumpta utcunque $t$ erit $tz^{\theta+4n-1} =$ Ordinatæ Curvæ cujus Area eſt $\frac{tz^{\theta}R^{\lambda} - \theta tg\text{C} - 2\lambda ntg\text{C} - \theta ht\text{D} - 3\lambda nht\text{D}}{\theta k + 4\lambda nk}$, & par eſt Ratio procedendi in Multinomiis omnibus ad Infinitum, quæ ſub formulis ſeu Theorematis hiſce Infinitis cadunt, quæque Geometricam Quadraturam non patiuntur. Et duabus quibuſcunque datis (vel ubicunque ſumptis) tam regredi quam progredi licet quouſque libuerit. Sed reſtat Caſus Secundus qui hujuſmodi eſt.

*Caſ.* 2. Sint jam Ordinatæ $pz^{\theta-1}R^{\lambda}$. $qz^{\theta+n-1}R^{\lambda}$ quibus reſpondent $p$A & $q$B Areæ, ſcil. $pz^{\theta-1}R^{\lambda}$ & $qz^{\theta+n-1}R^{\lambda}$, ducatur unaquæque in R & ad R rurſus applicetur, & habebis

R) $pz^{\theta-1}R^{\lambda+1}$ ($pz^{\theta-1}R^{\lambda} = z^{\theta-1}R^{\lambda-1} \times pe + pfz^{n} + pgz^{2n} + phz^{3n}$, &c.

R) $qz^{\theta+n-1}R^{\lambda+1}$ ($qz^{\theta+n-1}R^{\lambda} = qz^{\theta-1}R^{\lambda-1} \times qe + qfz^{n} + qgz^{2n}$, &c.

Aſſumatur e Prop. Tertia *Neutoniana* Areæ & Ordinatæ Mediantes.

| Ar. | Ord. |
|---|---|
| $az^{\theta}R^{\lambda}$ . . . . . | $\overline{\theta ae + \theta af + \lambda naf}\, z^{n} + \left.\begin{array}{l}\theta ag\\ 2\lambda nag\end{array}\right\} z^{2n} \times z^{\theta-1} R^{\lambda-1}$ |
| $bz^{\theta+n}R^{\lambda}$ . . . . . . . | $\overline{\theta be + nbe}\, z^{n} + \left.\begin{array}{l}\theta bf\\ nbf\\ \lambda nbf\end{array}\right\} z^{2n} \times z^{\theta-1} R^{\lambda-1}$, &c. |

& harum quatuor Arearum ſumma eſt $p\text{A} + q\text{B} + az^{\theta}R^{\lambda} + bz^{\theta+n}R^{\lambda}$ & ſumma Ordinatarum reſpondentium

$\theta ae$

$$\begin{array}{r}\theta ae+\\ pe+\end{array} \left.\begin{array}{r}\theta af\\ \lambda naf\\ \theta be\\ pf\\ qe\end{array}\right\} z^{n} \quad \left.\begin{array}{r}\theta ag\\ 2\lambda nag\\ \theta bf\\ \lambda nbf\\ pg\\ qf\end{array}\right\} z^{2n} \quad \left.\begin{array}{r}\theta bg\\ nbg\\ 2\lambda nbg\\ qg\end{array}\right\} z^{3n}\times z^{\theta-1}\ R^{\lambda-1}$$

ponatur primus Terminus $=0$ ſeu $\theta ae + pe = 0$ & $\theta ae = -pe$ & $\theta a = -p$. Unde

2°. $\theta af + \lambda naf + \theta be + nbe - \theta af + qe = 0$. Unde evaneſcit $p$ in Definitione $q$) Unde

3°. $\theta ag + 2\lambda nag + \theta bf + nbf + \lambda nbf + pg + qf = 0$. Unde rurſus evaneſcit $p$ in Def. $q$. Unde

4°. $\theta bg + nbg + 2\lambda nbg + qg = 0$. Et $qg = -\theta b - nb - 2\lambda nb$. Inde perterium eliminando ($p$ & $q$)

$\theta ag + 2\lambda nag + \theta bf + nbf + \lambda nbf - \theta ag - \theta bf - nbf - 2\lambda nbf = 0$, ſeu $2\lambda nag - \lambda nbf = 0$, & $2\lambda nag = \lambda nbf$ ſeu $2ag = bf$ & $\frac{2ag}{f} = b$. Unde ſecundus, ſcil. $\theta af + \lambda naf + \theta be + nbe + pf + qe$ evadit $\theta af + \lambda naf + \frac{2\theta age}{f} - \theta af - \frac{2\theta age}{f} - \frac{2nage}{f} - \frac{4\lambda nage}{f} = \frac{\lambda naff - 4\lambda nage}{f} =$ ſecundo Termino; adeoque ſumma quatuor Ordinatarum eſt $\frac{\lambda naff - 4\lambda nage}{f} z^{\theta+n-1} R^{\lambda-1}$, & ſumma totidem reſpondentium Arearum eſt $az^{\theta} R^{\lambda} + \frac{2ag}{f} z^{\theta+n} R^{\lambda} - \frac{\theta a A - 2^{\theta} + 2^{n} + 4^{\lambda n}}{f} agB$ ( eſt enim $q = -\frac{2\theta ag}{f} - \frac{2nag}{f} - \frac{4\lambda nag}{f}$) dividantur hæ ſummæ per $\frac{\lambda naff + 4\lambda nage,}{f}$ ſcil. per Coefficientem ſummæ Ordinatarum, & ſi quotum poſterius dicatur D erit D Area Curvæ cujus Ordinata eſt quotum prius $z^{\theta+n-1} R^{\lambda-1}$, &c. Q. E. I.

Jam pro Ordinata $z^{\theta-1} R^{\lambda-1}$

$\left.\begin{array}{r} -\theta b - nb - 2\lambda nb = q\\ \frac{\theta ag + 2\lambda nag + pg}{\lambda nf} = b\end{array}\right\}$ tum exterminando inventas $b$ & $q$ in ſecundo

Termino invenitur $p$ quæ ſubſtituenda eſt in primo Termino, ſcil. $\theta af + \lambda naf + \frac{\theta ag + 2\lambda nag + pg}{\lambda nf} \times \theta e + ne \left(= b \times \overline{\theta e + ne} = \theta be + nbe\right) + pf - \theta be - nbe - 2\lambda nbe = 0$ ſeu $\theta af + \lambda naf + pf - 2\lambda nbe = 0$, & ſubſtituendo $b$ in $2\lambda nbe$ habebis $\frac{\theta aff + \lambda naff - 2\theta age - \lambda nage - f}{2ge - ff} = p$ & $\frac{\lambda naffe - 4\lambda nagee - fe}{2ge - ff} = pe + \theta ae$. Unde poſitis ut ante $-\theta b - nb - 2\lambda nb = q$ & $\frac{\theta ag + 2\lambda nag + pg}{\lambda nf} = b$, & $\frac{\theta aff + \lambda naff - 2\theta age - 4\lambda nage - f}{2ge - ff} = p$. Erit 1. $\frac{\lambda naffe - 4\lambda nagee - fe}{2ge - ff} z^{\theta - 1} R^{\lambda - 1} =$ Ordinatæ. Et $az^{\theta} R^{\lambda} + \frac{\theta ag + 2\lambda nag + pg}{\lambda nf} z^{\theta + n} R^{\lambda} - \frac{\theta aff + \lambda naff - 2\theta age - 4\lambda nage - f}{2ge - ff} \times A$ $\left(-\overline{\theta b - nb - 2\lambda nb}\, B =\right) \frac{\theta ag + 2\lambda nag + pg}{\lambda nf} \times \overline{\theta - n - 2\lambda n} \times B$; Et dividendo utrinque ſummas haſce inventas

$$\frac{az^{\theta}R^{\lambda} + \frac{\theta ag + 2\lambda nag + pg}{\lambda^{n}f} z^{\theta + n}R^{\lambda} - \frac{\theta aff + \lambda naff - 2\theta age - 4\lambda nage - f}{2ge - ff} A - \frac{\theta ag + 2\lambda nag + pg}{\lambda^{n}f} \times \frac{\theta - n - 2\lambda n}{\lambda^{n}f} B}{\frac{\lambda naffe - 4\lambda nagee - fe}{2ge - ff}} =$$

$C =$ Areæ, cujus Ordinata eſt $z^{\theta - 1} R^{\lambda - 1}$.

## *SCHOLIUM.*

Conſimili prorſus proceſſu agendum eſt in aliis Caſibus harum Propoſitionum & patet etiam aditus ad Propoſitiones ulteriores quæ in hujuſmodi Seriebus pergunt ad Infinitum. Uti $pz^{\theta - 1} R^{\lambda}$ & $z^{\theta + n - 1} R^{\lambda}$ quibus reſpondebant $pA$ & $qB$ evaſerunt $pe + pfz^{n} + pgz^{2n} \times z^{\theta - 1} R^{\lambda - 1}$ & $qez^{n} + qfz^{2n}$, &c. $\times z^{\theta - 1} R^{\lambda - 1}$, ita $rz^{\theta - 1} R^{\lambda - 1}$ & $sz^{\theta + n - 1} R^{\lambda - 1}$, quibus reſpondent $rD$ & $sC$, ſi in $e + fz^{n}$, &c. ut antea ducantur & ad eandem applicentur evadent $\overset{(\alpha)}{r}\,z^{\theta-1}R^{\lambda-2}\times re+rfz^{n}+rgz^{2n}$, &c. & $\overset{(\beta)}{s}\,qz^{\theta+n-1}R^{\lambda-2}\,sez^{n}+\times sfz^{2n}$, &c. ita $\alpha z^{\theta-1}R^{\lambda-2}$, quibus reſpondebant, &c. $\alpha$E & $\beta$F ſi ut antea ducantur & applicentur, evadent $\alpha z^{\theta-1}R^{\lambda-3}$ & $\beta z^{\theta+n-1}R^{\lambda-3}$, &c. ad Infinitum; & hujuſmodi proceſſus in Prop Octava & cæteris quæ formari conſimili modo poſſint, continuantur ad Infinitum.

D. *Cheyne* Loco Arearum, Fluentes Fluxionum datarum inter ſe comparat ut ſequitur. *Vid.* pag. 94.

*Caſ.* 1. Ubi Fluxiones datæ eadem indeterminata exprimuntur, & nullis clauduntur Vinculis aut Radicalitatibus inſtituitur comparatio per Vulgarem Diviſionem, *v. g.* F : $x^{m}\dot{x}$ eſt ad F : $x^{n}\dot{x}$ eſt $\frac{n+1}{m+1}x^{m-n}$.

*Caſ.* 2. Ubi Fluxiones datæ eadem indeterminata exprimuntur, & iiſdem Vinculis, iiſdemque Quantitatibus ſub Vinculis comprehenſis gaudent, ponit alteram Fluxionem $=\dot{A}$ comparandam cum altera $\dot{B}$, & Canonem pro Fluente A per Methodos antea traditas inveniendum $=$ C & ultimi Termini in eo Canone abrumpentis Coefficientem $=$Q; item Canonem pro Fluenti ipſius $\dot{B}=$ D & Termini ultimo abrumdentis Coefficientem $=$ P; eritque, inquit, ſemper $A=mQB+C$ & $B=nPA+D$. Adeoque $A=\frac{mQB+C}{1-mnPQ}$, & $B=\frac{nPC+D}{1-mnPQ}$; itemq; $\frac{A}{B}=\frac{mQD+C}{nPC+D}$. Ubi $m$ & $n$ ex dato Caſu quocunqve ſunt Quantitates datæ.

*Caſ.* 3. Ubi Fluxiones datæ eadem indeterminata exprimuntur, ſed aut diverſis Vinculis, aut diverſis Quantitatibus ſub Vinculo comprehenſis, vel & diverſis Vinculis & diverſis Quantitatibus ſub Vinculo comprehenſis gaudent, inſtitui poterit per *Caſ.* precedentem utriuſque Fluentium Comparatio cum Fluente Fluxionis ortæ ex producto Quantitatum per Vincula ſignificatarum in Fluxionem indeterminatæ, adeoqne & ſic haberi poterit alterius ad alteram relatio. Verum hic non procedunt Conditiones finitæ rationis talium Fluentium ni res degeneret in Caſum ſecundum aut primum.

CAP.

# CAP. XII.

## *De Transformatione Curvarum.*

PRoximum *Neutoni* Principium pro Quadraturis promovendis ratione nititur e ſexto Elementorum petita, ex Conſideratione quod ea Rectangula ſint Æqualia quorum Latera reciproce proportionalia ſunt, unde, ſcil. æquantur Curvarum Areæ quarum Ordinatæ ſunt reciproce ut Fluxiones Abſciſſarum. Quia ſcil, Elementa Curvarum ſeu Fluxiones Arearum quaſi talia conſtituant Rectangula, cum Ordinatæ inter ſe parallelæ ſint, & Axi ad Angulos rectos inſiſtant, & hinc quidem exurgit Fundamentum Comparationibus Innumeris e quibus quaſdam in Corollariis ſequentibus explicat. Sit *e. g.* 1. Ordinata $z^{\theta-1} \times \overline{e+fz^{n}+gz^{2n}, \&c.}|^{\lambda}$. Et ſumpta Quantitate quacunque pro $\nu$ factaque $z^{\frac{n}{\nu}} = x$, erit $z^{\nu} = x^{\nu}$ unde $n\dot{z}z^{n-1} = \nu\dot{x}x^{\nu-1}$ & $\dot{z} = \frac{\nu\dot{x}x^{\nu-1}}{nz^{n-1}}$ & (ob $x^{\nu-1} = z^{n-1}$) erit quoque $\dot{z} = \frac{\nu\dot{x}}{n}$. Sed $\dot{x} : \dot{z} :: z^{\theta-1} : \frac{\dot{z}z^{\theta-1}}{\dot{x}}$ & $z^{\theta-1} = x^{\frac{\nu\theta-n}{n}}$. Unde $\dot{z}z^{\theta-1} = \frac{\frac{\nu\dot{x}}{n}x^{\frac{\nu\theta-n}{n}}}{\dot{x}} = \frac{\nu}{n}z^{\frac{\nu\theta-n}{n}}$, &c. & *Coroll.* 3 & 4. eodem modo ſequuntur. Sit jam in *Coroll.* 5. $z^{\theta-1} \times \overline{e+fz^{n}+gz^{2n}}|^{\lambda}$

fac $\frac{1}{z} = x = z^{-1}$
erit $x^{\theta} = z^{-\theta}$
ſeu $x^{-\theta} = z^{\theta}$
& $x^{-\theta-1} = z^{\theta-1} = x^{\frac{1}{\theta+1}}$

Eodem modo rurſus quia

$$\frac{1}{z} = x = z^{-1}$$

ſeu $\frac{1}{z} = x$ erit $1 = zx$

& $\frac{1}{x} = x^{-1} = z$

& $x^{-n} = z^{n}$ unde

$$z^{\theta-1} \times \overline{e + fz^{n} + gz^{2n}}|^{\lambda} = \frac{1}{x^{\theta+1}} \times \overline{e + fx^{-n} + gx^{-2n}}|^{\lambda} = \frac{1}{x^{\theta+1+2\lambda n}} \times$$

$\overline{g + fx^{n} + ex^{2n}}|^{\lambda}$. Eodem modo $z^{\theta-1} \times \overline{e + fz^{n} + gz^{2n} + hz^{3n}}|^{\lambda} = \frac{1}{x^{\theta+1}}$ $\times \overline{e + fx^{-n} + gx^{-2n} + hx^{-3n}}|^{\lambda} = \frac{1}{x^{\theta+1+3\lambda n}} \times \overline{e + fx^{n} + gx^{2n} + hz^{3n}}|^{\lambda}$ &c. & ſic pro cæteris. Ut jam data quavis Fluxione, ſeu Curvæ Elemento, proponatur eandem transformare in aliam quamcunque, quæ ſit Elementum ſeu Fluxio Areæ, Curvæ alterius, priori æqualis.

*Aſſumenda eſt Indeterminata nova x in data quavis ratione ad Priorem z (quæ ſupponitur eſſe Axis in Tranſmutanda data;) & aſſumpta hæc x erit Axis in Curva quæ quæritur. Jam*

1°. Inveniantur Valores $z$ & $y$ in Terminis $x$ expreſſi.

2°. Sumatur Fluxio Novæ Abſciſſæ $x$.

3°. Inſtituatur Proportio cujus primus Terminus ſit Fluxio Novæ Abſciſſæ; Fluxiones primæ Abſciſſæ & Ordinatæ expreſſæ per Novam $z$ ſint Termini ſecundus & tertius, & quartus Terminus hinc Oriundus erit Ordinata tranſmutatæ; & Rectangulum extremorum erit Elementum Areæ tranſmutatæ, & Rectangulum ſeu factum a mediis erit Elementum Curvæ propoſitæ.

Sit jam ſecunda illa Ordinata in prima Tabula *Neutoniana*, viz. $\frac{dz^{n-1}}{e+fz^{n}}$, ut tranſmutetur hæcce jam Curva in aliam priori æqualem, cujuſque Axis erit $x$. Aſſume $x = z^{n}$. Inde autem $z = x^{\frac{1}{n}}$, & $z^{n-1} = x^{1-\frac{1}{n}}$. $\dot{x} = \frac{1}{n}\dot{z}z^{n-1}$ & $\frac{dz^{n-1}}{e+fz^{n}} = \frac{dx^{1-\frac{1}{n}}}{e+fx}$. Jam inſtituenda eſt proportio $\dot{x} : \dot{z}\ (= \frac{1}{n}x^{\frac{1}{n}-1}\dot{x}) :: \frac{dz^{n-1}}{e+fz^{n}}\ (= \frac{dx^{1-\frac{1}{n}}}{e+fx}) : \frac{d}{n \times \overline{e+fx^{n}}}$ Quartus hic Terminus erit Ordinata tranſmutatæ, cum Elementum, ſcil. factum ab Extremis æquatur facto a Mediis ; & Rectangulum e mediis eſt Elementum Curvæ propoſitæ ; & ejuſdem Area æqualis erit Areæ propoſitæ ſeu facto ab extremis.

Erit itaque F : $\frac{d\dot{x}}{n\times e+fx}$ ſeu Area Tranſmutatæ, æqualis Areæ propoſitæ, *viz.* Areæ Curvæ cujus Ordinata eſt $\frac{dz^{n-1}}{e+fz^n}$. Hoc brevius fit aliquando ſubſtituendo pro primo Termino proportionis, Fluxionem $\dot{x}$ expreſſam per $z$, pro ſecundo & tertio $\dot{z}$ & $\frac{dz^{n-1}}{e+fz^n}$ & quartus inde proveniens erit Ordinata transformatæ, poſtea exprimendæ per $x$. In Exemplo ultimo ubi $z^n = x$ Proportio erit $\dot{x} = n\dot{z}z^{n-1} : \dot{z} :: \frac{dz^{n-1}}{e+fz^n} : \frac{d}{n\times e+fz^n}$ & ſubſtituendo $x$ loco $z^n$ Ordinata Transformatæ erit $\frac{d}{n\times e+fx}$. Et Elementum ejuſdem Areæ erit $\frac{d\dot{x}}{n\times e+fx^n}$. Sed $\frac{d\dot{x}}{n\times e+fx} = \frac{1}{n}\times\frac{d\dot{x}}{e+fx}$ (& ſupponendo quod $d$ ſit Quadratum in Hyperbola Æquilatera, *i. e.* quod $d =$ *AG x KG, & quod $ef = 1$, $e =$ KF, $x =$ FI) eſt Elementum Spatii Quadrilateri Hyperbolici FI*if* ducti in $\frac{1}{n}$, & $\frac{1}{n} \times$ F : $\frac{dx}{e+fx}$ eſt Quadrilaterum illum ſuppoſitum tanquam datum. Hoc modo Fluens Fluxionis $\frac{dz^{n-1}\dot{z}}{e+fz^n}$ æqualis eſt Spatio Hyperbolico inter Aſymptotos FI*if* Hyperbolæ æquilateræ ducto in $\frac{1}{n}$. Et ſi animus ſit Lectori, poſſit Fluentes Fluxionum $\frac{dz^{2n-1}\dot{z}}{e+fz^n}$, $\frac{dz^{3n-1}\dot{z}}{e+fz^n}$, &c. ex hac data invenire per Caput precædens, & inventas more *Nentoniano* in Tabulas referre.

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* *Vid. Schem. Notat. ſub Calcem annexum.*

Et eodem modo pro $dz^{\frac{1}{2}n-1}\times\overline{e+fz^n}^{\,-1}$, &c. pro cedere licet ponendo ſcil. $\sqrt{\frac{d}{e+fz^n}} = x$, &c. ad Infinitum.

Sit $\frac{d}{z}\dot{z}\sqrt{e+fz^n}$ Elementum Curvæ, cujus Ordinata $\frac{d}{z}\sqrt{e+fz^n}$; Hoc dependet ex Quadratura Hyperbolæ vel Ellipſeos, ſeu ad F: $-\dot{x}\sqrt{f+exx}$, ſupponendo $z^{\frac{1}{n}}$ ſeu $z^{-n} = xx$. Unde $z^{-\frac{1}{2}n} = x$. $x^{\frac{1}{2}n} = x^{-1}$, & $z^n = x^{-2}$, & $-\frac{1}{2}nz^{-\frac{1}{2}-1}\dot{z} = \dot{x}$. Tum $\dot{x}\,(-\frac{1}{2}nz^{-\frac{1}{2}n-1}\dot{x}:)$ $\dot{z} : dz^{-1}\times\sqrt{e+fz^n} : \frac{2d}{-n}z^{\frac{1}{2}n}\sqrt{e+fz^n}$, & ſubſtituendo $x$ pro $z$ in Termino noviſſimo, habebitur $\frac{2d}{-n}x^{-2}\times\sqrt{f+exx}$; & multiplicando per $\dot{x}$ habebis pro tranſmutata $\frac{2d}{-n}x^{-2}\dot{x}\times\sqrt{f+exx}$. Et ſupponendo tanquam cognitam F: $-\dot{x}\times\sqrt{f+ex^2}$ invenietur per Problema præcedens Fluens $\frac{2a}{-n}x^{-2}\dot{x}\sqrt{f+ex^2}$ Fluens, ſcil. Fluxionis propoſitæ.

Ponatur jam tranſmutare $\frac{d\dot{z}}{ff+zz}$ in hanc, viz. $-\frac{d\dot{x}}{2\sqrt{x-ffx^2}}$ (quæ Elementum eſt Sectoris Circuli ubi $ff$ ſumitur pro Unitate arbitraria quæ eſt etiam Circuli Diameter) faciendo $z = \sqrt{\frac{1}{x}-ff}$; unde $z^2 = \frac{1}{x}-ff$ & $zz+ff = \frac{1}{x}$ & $\overline{zz+ff}|^2 = \frac{1}{x^2}.\ \frac{1}{x} = \frac{1}{zz+ff} = \overline{zz+ff}|^{-1}$ &

$$xx = \overline{zz+ff}|^{-2} \ \& \ \dot{x} = -\frac{2z\dot{z}}{\overline{zz+ff}|^2}.$$

$$\text{Tum } \dot{x}\left(-\frac{2z\dot{z}}{\overline{zz+ff}|^2}\right) : \dot{z} :: \frac{d}{ff+zz} : \left.\frac{\frac{d\dot{z}}{ff+zz}}{\frac{-2z\dot{z}}{\overline{zz+ff}|^2}}\right\} = \frac{d\dot{z}\times\overline{zz+ff}|^2}{2ffz\dot{z}+2z^3\dot{z}}$$

=

$= \frac{d \times \overline{zz + ff}|^2}{2z \times \overline{ff + zz}} =$ (ſubſtituendo Valorem $z$) $- \frac{d}{2\sqrt{x - ffxx}}$ quoniam

$$\frac{-\frac{d}{x}}{2\sqrt{\frac{1}{x} - ff}} = - \frac{d}{2x\sqrt{\frac{1}{x} - ff}} = - \frac{d}{2\sqrt{xx}\sqrt{\frac{1}{x} - ff}} = - \frac{d}{2\sqrt{\frac{xx}{x} - ffxx}}$$

$= - \frac{d}{2\sqrt{x - ffxx}}$ & multiplicando quartum hunc Terminum per primum $\dot{x}$ habebitur $- \frac{d\dot{x}}{2\sqrt{x - ffxx}} =$ Elemento Sectoris Circuli ducto in conſtantem $- a$. Quod & adhuc ducendus eſt in $\frac{1}{4}$ ex Hypotheſi quod $f$ ſit tam Unitas arbitraria quam Circuli Diameter.

Sit denique Curvæ illius (olim a *Leibnitio* ad *Oldenburgum* propoſitæ) Elementum $- \frac{8r^5xx\dot{x}}{\overline{rr + xx}|^3}$. Ponatur $xx = \frac{2r^3 - rru}{u}$ erit $xxu = 2r^3 - rru$ & (capiendo Fluxiones) $2x\dot{x}u + xx\dot{u} = - rr\dot{u}$ ſeu $2x\dot{x}u = - rr\dot{u} - xx\dot{u}$

& $\dot{x} = - \frac{rr\dot{u} - xx\dot{u}}{2u\frac{r}{u}\sqrt{2ru - uu} = 2xu}$ ſeu $\dot{x} = \frac{\overline{rr - xx} \times \dot{u}}{\frac{2ru}{u}\sqrt{2ru - uu}} = \frac{\overline{rr - xx}\dot{u}}{2r\sqrt{2ru - uu}}$.

Et $\dot{x} = \frac{\frac{rr\dot{u}}{1} - \frac{2r^3\dot{u} - rru\dot{u}}{u}}{2r\sqrt{2ru - uu}} = \frac{\frac{rru\dot{u} - 2r^3\dot{u} - rru\dot{u}}{u}}{2r\sqrt{2ru - uu}} = - \frac{2r^3\dot{u}}{2ru\sqrt{2ru - uu}}$

$= - \frac{rr\dot{u}}{u\sqrt{2ru - uu}}$; & inſtituendo proportionem $\dot{u} : \dot{x}$ $(= - \frac{ru\dot{u}}{u\sqrt{2r - uu}})$

$:: - \frac{8r^5x^2}{\overline{rr + xx}|^3}$ $(- \frac{2ruu + u^3}{rr})$: $\frac{2ru - uu}{\sqrt{2ru - uu}} = \sqrt{2ru - uu}$, & habebis $\dot{u}\sqrt{2ru - uu}$ pro Fluxione transformatæ ſeu tranſmutatæ; & $F : \dot{u}\sqrt{2ru - uu}$ exprimet Aream Circuli.

# CAP. XIII. *ſeu* CONCLUSIO.

## *Synopſis ſeu Syllabus Summatorius Inventionum in præcedentibus exhibitarum,* &c.

UT jam Concluſionis loco ſummatim agamus; ex præcedentibus ſatis, credo, patebit celeberrimum *Neutonum* primum extitiſſe hujuſce Methodi (quocunque demum nomine appelletur) Authorem; & *Leibnitium* non niſi longo poſt temporis intervallo in eundem incidiſſe, ſiquidem id a Commercio quod habuerit Epiſtolico cum *Neutono* non mutuatus fuerit; quod ſemper ſuſpicati ſunt non pauci, Dum vero hæc ſub prælo eſſent, & magna ex parte impreſſa, prodiit Liber *Societatis Regiæ juſſu* Commercium Epiſtolicum D. *Johannis Collins*, & aliorum de Analyſi promota continens. Ubi tota Controverſia de Methodi ſive *Fluxionum* ſive *Differentialis* Authore ex *Neutoni & Leibnitii* autographis fuſius & clarius explicatur. Loco ergo eorum quæ hiſce prius ſubnexa habuiſſemus eo tendentia, ex eo ipſo Libro breviter delibata congeremus; ex quo ergo conſtat *Neutonum* ante Annos 64 & 65 hanc Methodum inveniſſe, & extractionem Radicum in Speciebus, & etiam Fluxiones omnes exinde ad Infinit. primas, ſecundas, &c. deſignaſſe hoc modo, viz, per $\overline{x+o}|^{\frac{m}{n}}$ in Seriem

$$x^{\frac{m}{n}} + \frac{m}{n} ox^{\frac{m-n}{n}} + \frac{m^2 - mn}{2n^2} oox^{\frac{m-2n}{n}} + \frac{m^3 - 3m^2n - 2mn^2}{6n^3} ooox^{\frac{m-3n}{n}}, \text{ \&c.}$$

converſam uti in Epiſtola prima ad *Oldenburg.* & Fluxionum Scholio ſub calcem Quadraturum annexo videre eſt. Menſe Octobris 1676. primam harum Epiſtolam conſcripſit cum *Leibnitio* communicandam dat 13 Jun. & alteram 24 Octobris ejuſdem Anni. In prima præter extractionem Radicum in Seriebus, Quadraturam Quadratricis (Methodum Summatoriam uti *Leibnitius* loquitur plane involventem) exhibibet. In ſecundo inventionem Fluxionum & Fluentium, & etiam Methodos duas pro Æquationibus Fluxionalibus, literis tranſpoſitis (uti initio hujus tractatus memoravimus) celatam aliaque multa miſit; & Exempla Quadraturarum addit, ut & Comparationes Curvarum jamdudum ſe habuiſſe refert, *p.* 76. & *p.* 79. Modeſte *Leibnitium* parum animadvertiſſe meminit *quod dum poſuit Seriem pro Determinatione coſinus ex Arcu dato, vix animadvertiſſe Seriem ſuam (quam paulo ante miſerat) pro Determinatione Sinus verſi ex eodem Arcu; ſiquidem hæc idem ſunt.* Pro regreſſione ab Areis ad Lineas rectas & ſimilibus duo (inter cætera quæ habuit) Thoremata proponit; aſt *alterum modum regrediendi ab Areis ad Lineas rectas* (quem habuit) *cclare ſtatuit.* Hæc autem erit ipſa Methodus Fluxionum directa: & tunc dicit *Inverſa de Tangentibus Problemata eſſe in Poteſtate, aliaque illis difficiliora*, per Methodos, ſcil. Fluxionales quas tum celatis literis ibidem conſignavit aptis (ſi qui eſſent) interpretibus. *Leibnitius* autem (ut ibidem conſtat anno 167$\frac{6}{7}$ aliam Differentiarum Methodum habuit; & Anno 1670 circiter cæpit *Collinius* Series *Neutonianas* & 1671 *Gregorianas* cum Amicis communicare. *Leibnitius* Anno 1673 *Londini* erat ſed nullas Series ibidem niſi quas acceperat ab *Oldenburgo* communicavit; ad quem a Pariſiis 30 Martii 1675 ſcribit, ſe ab eo intellexiſſe *Neutonum Quadraturas omnes, omniumque Curvarum Superficierum & Solidorum ex revolutione genitorum Dimenſiones & Centrorum Gravitates inventiones per approximationes attigiſſe. Quæ Methodus, inquit, ſi eſt Univerſalis & Commoda meretur æſtimari*; nec dubitat quin ſit *Ingenioſiſſimo Authore digna.* Neque de ſuis in hoc genere Inventionibus huc uſque ulla injicitur Mentio. 20 Maii 1675 loquitur de Seriebus quibuſdam a *Collinio* accptis ut cum ſuis compararentur. Et eodem *Anno* 1675 compoſitum habebat Opuſculum *Quadraturæ Arithmeticæ*; quæ; inquit Commentator in *Commercium Epiſtolicum, ea eſt* Gregorius *cum D.* Collinio *initio Anni* 1671, Oldenburgus *cum* Leibnitio *hoc Anno Communicavit.* Et *de hac Quadratura*, pergit, D. *Leibnitius opuſculum Vulgari More compoſuit & cum Amicis hoc Anno communicare cæpit: Anno proximo ſcriptum polivit ut cum* Oldenburgo *communicaretur: Anno tertio in Patriam redux Negotiis Publicis interceſſe cæpit, & materia ſub Manibus creſcente Opus ad Editionum limare non amplius vacavit. Sed neque Operæ pretium duxit ſubinde prolixius exponere vulgari more quæ Analyſis ſua nova paucis exhibet. Inventa eſt igitur hæc Analyſis poſtquam D.* Leibnitius *opuſculum vulgari more compoſitum polire & limare deſiit, & Negotiis Publicis interceſſe cæpit.*

Anno ergo 1677 imprimis *Leibnitius* Methodum ſuam differentialem cum Amicis communicare cæpit, licet a multo tempore, ut ait, eam habuit, & rem Tangentium tractaverat per differentias Ordinatarum; at Epiſtolas etiam duas *Neutoni* prædictas perlegerat. Sed tamen in

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*Vid. p.* 38, 89. 117.

Epiſtola 27 Auguſt. 1676. Problema ibi aggreſſus Methodum *prolixiori* More exponit, quam *paucis* (ut animadvertit Commentator) Analyſi ſua nova, *modo tunc temporis habuiſſet* exhibuiſſe potuiſſet; unde Merito concludit quod eandem nondum habuit; atque itidem rogat, *Quomodo in Methodo regreſſuum ſe gerat, ut cum ex Logarithmo quærit numerum*, &c. *& exoptat Demonſtrationem approximationis* Gregorianæ. Et ait (ibid.) *Multa uſque adeo mira & implexa eſſe ut neque ab Æquationibus pendeant neque ex Quadraturis. Qualia ſunt (ex multis aliis) Problemata Methodi Tangentium inverſæ; quæ etiam* Carteſius *in poteſtate non eſſe faſſus eſt.* Reſcripſit autem *Neutonus, Hujuſmodi Problema in Poteſtate eſſe*, per Æquationes ſuas *& tum demum* (uti Annotator obſervat) *Lebnitius* a *Neutono admonitus hæc vidit. Et ſi Æquationes, pergit, differentiales* D. Leibnitio *jam tum innotuiſſent haud dixiſſet Problemata Methodi Tangentium inverſæ ab Æquationibus non pendere.*

12° Maii 1676. accepiſſet per Dom. quendam *Georgium Mohr* Danum a *Collinio* communicatas duas Series *Neutoni* Expreſſionem exhibentes relationis inter Arcum & Sinum per Infinitas Series ut ſequitur, viz. poſito Sinu $= x$, Arcu $= z$, Radio $= 1$, erunt

$$z = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9, \&c.$$

$$x = z - \frac{1}{6}z^3 + \frac{1}{120}z^5 - \frac{1}{5040}z^7 + \frac{1}{362880}z^9, \&c.$$

Hæc illi uti dicit valde videbantur ingenioſa & Demonſtrationem petit, & poſtea Methodum perveniendi ad has Series, ſuo rogatu a *Neutono* acceperat una cum Seriebus ipſis *& Ægre intellexerat, & poſtea tamen eandem inter Chartas ſuas antiquas invenit*; Sed & ante hoc tempus hujuſmodi Artes ei in uſu erant; Seriem enim quandam inter alias a *Gregorio* inventam ad Tangentes naturales ex earundem Arcubus, & converſim obtinendas, viz. poſitis Radio $= r$, Arcu $= a$, Tangente $= t$;

erit $t = a + \frac{a^3}{3r^2} + \frac{2a^5}{15r^4} + \frac{17a^7}{315r^6} \; \frac{62a^9}{2837r^8} +$ &c. & converſim ex Tangente invenire Arcum ejus $a = t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4} - \frac{t^7}{7r^6} + \frac{t^9}{9r^8} -$ &c.

*Hanc Seriem* (inquit Commentator) *initio Anni* 1671. *a* Gregorio *acceperat, & D.* Leibnitius *eandem cum amicis in* Gallia *hoc Anno ut ſuam communicavit, celata* Oldenburgi *Epiſtola in qua eandem acceperat.*

Per

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*Vid.* p. 63. 65. 45. 41.

Per plura hujuſmodi Artificia & Commercia apud tractatum prædictum invenire eſt, ſed pauca hæc hic ſufficiant; Addemus tantum Teſtimonium D. *Nicolai Fatii Duillierii* R. S. S. in diſſertatione de inveſtigatione Geometrica Lineæ breviſſimi deſcenſus, &c. *Londini* Anno 1699. Edita *p.* 18. ubi hæc leguntur.

Neutonum *primum, ac pluribus Annis Vetuſtiſſimum, hujus Calculi inventorem, ipſa rerum evidentia coactus, agnoſco: a quo ultrum quicquam mutuatus ſit* Leibnitius ſecundus *ejus Inventor, malo eorum, quam meum, ſit judicium, quibus viſæ fuerint* Neutoni *Literæ aliique ejuſdem Manuſcripti Codices*; & eodem tendit Sententia Societatis Regiæ, ad quam D. *Leibnitius* provocaverat, ad Calcem hujuſce commercii Epiſtolici annexa; quæque eſt ut ſequitur, occaſione nata, ex Vindicatione hujus Inventionis ſuo proprio Authori a D. *Johanne Keill* nunc Profeſſore apud *Oxonienſes*, &c.

*Literas & Literarum Apographa tam quæ* “ in Archivis Regiæ Societatis, quam quæ inter Chartas D. *Johannis Collinii* aſſervantur, & inter Annos 1669 & 1677, datæ ſunt inſpeximus; & ex his quæ D. *Barrovii*, D. *Collinii*, D. *Oldenburgi*, & D. *Leibnitii* Nomen ferebant, ex fide aliquorum qui eorum Autographa probe noverant, ipſorum eſſe certo didicimus. Literas autem quæ *Gregorium* præ ſe ferebant Autorem, ipſius eſſe cognovimus Fide *Collinii*, qui nonnullas earum *Gregorio* aſſignatas manu ſua exſcripſerat. Ex his omnibus excerpſimus quæcunque ad rem nobis commiſſam pertinere videntur; atque illa excerpta quæ una cum ipſis Literis jam vobis traduntur, fideliter & accurate facta eſſe comperimus. Ex his autem Literis Chartiſque nobis conſtat.

1. “ D. *Leibnitium* Anno ineunte 1673. *Londini* fuiſſe, unde Menſe *Martio* vel circiter *Pariſios* adiit, ubi Literarum Commercium habuit cum D. *Collinio* intercedente *Oldenburgo*, uſque in Menſem *Septembrem* 1676. Deinde per *Londinum* & *Amſtelodamum Hannoveram* reverſum eſſe. D. autem *Collinium* Matheſeos peritis ea quæ a D. *Neutono* & *Gregorio* acceperat lubentiſſime communicaſſe.

2. “ D. *Leibnitium*, cum prima vice *Londinum* adiit, Methodi cujuſdam Differentialis proprie ſic dictæ, ſe inventorem perhibuiſſe: Et etiamſi D. *Pellius* ipſi monſtraverat eandem antea a D. *Moutono* uſurpatam fuiſſe, haud tamen ſibi Inventoris jura aſſerere deſtitiſſe; cum quia proprio ut aiebat *Marte* ſua illa inveniſſet, nondum viſis quæ *Moutonus* prius ediderat, tum quia plurima adjeciſſet. Neque uſquam mentionem reperimus factam alterius Methodi ejus Differentialis præter iſtam *Moutoni* ante Literas ejus 21 *Junii* 1677 datas; hoc eſt Anno integro poſtquam D. *Neutoni* Epiſtola, 10 *Decembris* ſcripta, *Pariſios* ipſi communicanda tranſmiſſa fuit; and quadriennio poſtquam D. *Collinius* eandem Epiſtolam cum Amicis communicare

“ cæpit. In hac autem Epiſtola Methodus Fluxionum idoneo harum
“ Rerum cognitori *evidenter ſatis deſcribitur.*

3. “ Ex Literis D. *Neutoni* 13 *Junij* 1676 datis, manifeſtum eſt Fluxionum
“ Methodum ipſi innotuiſſe, quinquennio prius quam Epiſtolam
“ illam ſcriberet. Et ex *Analyſi* ejus per *Æquationes* numero Terminorum
“ infinitas, quam *Barrovius* cum D. *Collinio* Menſe *Julio* Anni
“ 1669 communicavit, conſtat illum etiam ante illud tempus eandem
“ excogitaſſe.

4. “ Methodus Differentialis una eademque eſt cum *Methodo Fluxionum*
“ ſi nomen & Notationis modum exceperis. D. *Leibnitius* enim
“ eas Quantitas *Differentias* appellat quas D. *Neutonus Momenta* vel
“ *Fluxionis:* Eaſque Nota Literæ [*d*] deſignat, quam non adhibet
“ D. *Neutonus.* Rem proinde de qua agimus hanc Autumamus eſſe;
“ non uter hanc uter illam Methodum invenerit; *ſed uter Methodum*
“ *ipſam, quæ unica eſt, prior invenerit* Simul illos qui D. *Leibnitium*
“ pro inventore primo habuete, de eo quod inter illum & D. *Collinium*
“ olim interceſſerat Commercio parum aut nihil reſciviſſe opinamur;
“ neque intellexiſſe D. *Neutonum* eadem Methodo uſum eſſe, quindecim
“ prius Annos quam D. *Leibnitius* eam in Actis Eruditorum *Lipſiæ*
“ evulgare cœpit.

“ Quibus perpenſis, D. *Neutono* primum eſſe hujus Methodi inventorem
“ arbitramur; atque ideo D. *Keillium*, eandem illi aſſerendo,
nullo modo D. *Leibnitium Calumnia aut injuria affeciſſe*, &c.” Hucuſque *Societas Regia.* Cæterum ut finem huic tractatui imponamus, unicuique conati ſumus, & Methodos etiam Tyronibus accommodatas exhibere.

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*Vid.* p. 17. 62. *pro Not.*

# F I N I S.

*Epiſtolæ ſequentes à* D. Leibnitio *cum amicis ſuis in Gallia & alibi communicatæ, ad controverſiam præcedentem ſpectant.*

*Prima ac tertia eſt ipſius* Leibnitii, *ſecunda* Newtoni, *omnes ad amicum communem.*

*Prima circa menſem Novembrem vel Decembrem Anni* 1715 *ſcripta, non eſt Epiſtola tota ſed Additamentum.*

### P. S.

“ VOILA’ Monſieur, la lettre dont vous pourrez faire
“ uſage ſi vous le jugez a propos. Je viens maintenant
“ à ce qui nous regarde. Je ſuis ravi que vous eſtes en
“ *Angleterre*, il y a dequoy profiter, & il faut avouer
“ qu’il y a là de trés habiles gens, mais ils vondroient paſſer pour étre
“ preſque ſeuls inventeurs ; & c’eſt en quoy apparemment ils ne reuſſiront
“ pas. Il ne paroiſt point que *M. Newton* ait eu avant moy la
“ Caracteriſtique & l’Algorithme infiniteſimal, ſuivant ce que M. *Bernoulli*
“ a tres bien jugé: Quoyqu’il luy auroit eté fort aiſé d’y parvenir
“ s’il s’en fut aviſé. Comme il auroit eté fort aiſé à *Apollonius* de
“ parvenir à l’Analyſe de Des Cartes ſur les Courbes, s’il s’en etoit
“ aviſé. Ceux qui ont ecrit contre moy n’ayant pas fait difficulté
“ d’attaquer ma candeur par des interpretations forcées & mal fondées ;
“ ils n’auront point le plaiſir de me voir répondre à de petites raiſons
“ de gens qui en uſent ſi mal, & qui d’ailleurs s’ecartent du fait. Il
“ s’agit du Calcul des differences, & ils ſe jettent ſur les Series, où
“ M. *Newton* m’a precedé ſans difficulté ; mais Je trouvay enfin une
“ Methode generale pour les Series, & aprés cela Je n’avoit plus beſoin
“ de recourir à ſes extractions. Ils auroient mieux fait de donner
“ les Lettres entieres comme M. *Wallis* a fait avec mon conſentement,
“ & il n’a pas eu la moindre diſpute avec moy, comme ces gens la
“ voudroient perſuader au Public. Mes Adverſaires n’ont publié du
“ Commercium Epiſtolicum de M. *Collins*, que ce qu’ils ont crû capable
“ de recevoir leur mauvaiſes Interpretations. Je fis connoiſſance avec
“ M. *Collins* dans mon ſecond Voyage d’*Angleterre*, car au premier (qui

“ dura tres peu parceque J'etois venu avec un Miniſtre public) Je
“ n'avois pas encore la moindre connoiſſance de la Geometrie avancée,
“ & n'avois rien vû ni entendu du commerce de M. *Collins* avec meſs.
“ *Gregory* & *Newton*; comme mes Lettres echangées avec M. *Oldenbourg*
“ en ce temps là, & quelque temps apres, feront aſſez voir. Ce n'eſt
“ qu'en *France* que J'y ſuis entré, & M. *Hugens* m'en donna l'entrée.
“ Mais a mon ſecond Voiage, M. *Collins* me fit voir une Partie de ſon
“ commerce, & J'y remarquay que M. *Newton* avoua auſſi ſon ignorance
“ ſur pluſieurs choſes, & dit entre autres qu'il n'avoit rien trouvé ſur
“ la Dimenſion des Curvilignes celebres que la Dimenſion de la Ciſſoide.
“ Mais on a ſupprimé tout cela. Je ſuis faché qu'un auſſi habile
“ homme que M. *Newton* s'eſt attiré la cenſure des Perſonnes Intelligentes,
“ en deferant trop aux ſuggeſtions de quelque flatteurs,
“ qui l'ont voulu brouiller avec moy.

“ Sa Philoſophie me paroit un peu étrange, & Je ne crois pas
“ qu'elle puiſſe s'etablir. Si tout corps eſt grave, il faut neceſſairement
“ (quoyque diſent ſes deffenſeurs & quelque emportement qu'ils
“ temoignent) que la gravité ſoit une qualité occulte ſcholaſtique, ou
“ l'effect d'un miracle. J'ay fait voir autrefois à M. *Bayle* que tout
“ ce qui n'eſt pas explicable par la nature des creatures eſt miraculeux.
“ Il ne ſuffit pas de dire, Dieu a fait une telle loy de Nature,
“ donc la choſe eſt naturelle. Il faut que la loy ſoit executable par
“ les natures des creatures. Si Dieu donnoir cette loy, par Exemple
“ à un corps libre, de tourner à l'entour d'un certain centre, il faudroit
“ ou qu'il y joignit d'autres corps qui par leur impulſion l'obligeaſſent
“ de reſter toujours dans ſon orbite circulaire, ou qu'il mit
“ une Ange à ſes trouſſes; on enfin il faudroit qu'il y concourut extraordinairement.
“ Car naturellement il s'ecartera par la Tangente. Dieu
“ agit continuellement ſur les Creatures par la conſervation de leur
“ Natures, & cette conſervation eſt une Production continuelle de ce
“ qui eſt Perfection en elles. Il eſt *intelligentia ſupramundana* parce qu'il
“ n'eſt pas l'ame du Monde, & n'a pas beſoin de ſenſorium.

“ Je ne trouve pas le vuide demonſtré par les raiſons de M. *Newton*
“ ou de ſes Sectateurs, non plus que la pretendue gravité univerſelle,
“ ou que les Atomes. On ne peut donner dans le vuide &
“ dans les Atomes, que par des vues trop bornées. M. *Clark* diſpute
“ contre le ſentiment des Carteſiens qui croyent que Dieu ne ſauroit
“ déſtruire une partie de la matiere pour faire un vuide, mais Je
“ m'etonne qu'il ne voye point que ſi l'eſpace eſt une ſubſtance differente
“ de Dieu, la même difficulté s'y trouve. Or de dire que Dieu eſt
“ l'eſpace, c'eſt luy donner des parties. L'eſpace eſt l'ordre des coexiſtences,
“ & le temps eſt l'ordre des exiſtences ſucceſſives. Ce ſont des
“ choſes veritables mais ideales, comme les nombres.

“ La matiere même n'eſt pas une ſubſtance, mais ſeulement *ſubſtantiatum*, un Phenomene bien fondé, & qui ne trompe point quand on y procede en raiſonnant ſuivant les loix ideales de l'Arithmetique de la Geometrie & de la Dynamique, *&c.* Tout ce que J'advance en cela paroiſt démontré. A propos de la Dynamique ou de la Doctrine des forces, Je m'etonne que M. *Newton* & ſes ſectateurs croyent que Dieu a ſi mal fait ſa Machine, que s'il n'y mettois la main extraordinairement, la Montre ceſſeroit bien tôt d'aller. C'eſt d'avoir des idées bien étroites de la ſageſſe & de la puiſſance de Dieu. J'appelle extraordinaire toute operation de Dieu, qui demande autre choſe que la conſervation des Natures des Creatures. Ainſi quoy que Je croye la Metaphyſique de ces Meſſieurs là, *a narrow one*, & leur Mathematique aſſez *arrivable*; le ne laiſſe pas d'eſtimer extremement les Meditations Phyſico-Mathematiques de M. *Newton*; & vous obligeriez infiniment le public, Monſieur, ſi vous portiez cet habile homme à nous donner juſqu'a ſes Conjectures en Phyſique. J'approuve fort ſa Methode de tirer des Phenomenes ce qu'on en peut tirer ſans rien ſuppoſer, quand même ce ne ſeroit quelquefois que tirer des Conſequences Conjecturales. Cependant quand les *data* ne ſuffiſent point, il eſt permis (comme on fait quelquefois en déchifrant) d'imaginer des Hypotheſes; & ſi elles ſont heureuſes on s'y tient proviſionellement, en attendant que des nouvelles Experiences nous apportent *nova Data*, & ce que *Bacon* apelle *experimenta crucis*, pour choiſir entre les Hypotheſes. Comme J'apprends que certain *Anglois* ont mal repreſenté ma Philoſophie dans leur Tranſactions, Je ne doute point qu'avec ce que Je vous mande icy, Je ne puiſſe etre juſtifié. Je ſuis fort pour la Philoſophie experimentale, mais M. *Newton* s'en e'carte fort quand il pretend que toute la matiere eſt peſante (ou que chaque partie de la matiere attire chaque autre partie) ce que les Experiences ne prouvent nulment, comme M. *Hugens* a deja fort bien jugé; la matiere gravifique ne ſauroit avoir elle même cette peſanteur dont elle eſt la cauſe, & M. *Newton* n'apporte aucune Experience ni raiſon ſuffiſante pour le vuide & les Atomes ou pour l'attraction mutuelle generale. Et parce qu'on ne ſait pas encore parfaitement & en detail comment ſe produit la gravité ou la force elaſtique, ou la magnetique, *&c.* On n'a pas raiſon pour cela d'en faire des Qualitez occultes Scholaſtiques ou des Miracles; mais on a encore moins raiſon de donner des bornes à la ſageſſe & à la puiſſance de Dieu, & de luy attribuer un *ſenſorium* & choſes ſemblables. Au reſte, Je m'etonne que les Sectateurs de M. *Newton* ne donnent rien qui marque que leur maiſtre leur a communiqué une bonne Methode, j'ay eté plus heureux en Diſciples.

Leiceſter-Fields,

S I R, London; Feb. 26, 17$\frac{15}{16}$.

YOU know that the *Commercium Epiſtolicum* contains the antient Letters and Papers preſerved in the Archives and Letter-Books of the Royal Society and Library of Mr. *Collins* relating to the Diſpute between Mr. *Leibnitz* and Doctor *Keill*, and that they were collected and publiſh'd by a numerous Committee of Gentlemen of ſeveral Nations appointed by the Royal Society for that purpoſe. Mr. *Leibnitz* has hitherto avoided returning an Anſwer to the ſame; for the Book is matter of Fact and uncapable of an Anſwer. To avoid anſwering it he pretended the firſt Year that he had not ſeen this Book, nor had Leiſure to examine it, but had deſir'd an eminent Mathematician to examine it. And the Anſwer of the Mathematician (or pretended Mathematician) dated *June* 7, 1713. was inſerted into a defamatory Letter dated *July* 29 following, and publiſh'd in *Germany* without the Name of the Author or Printer, or City where it was printed. And the whole has been ſince tranſlated into *French*, and inſerted into another abuſive Letter (of the ſame Author, as I ſuppoſe) and anſwer'd by Dr. *Keill* in *July*, 1714; and no Anſwer is yet given to the Doctor.

Hitherto Mr. *Leibnitz* avoided returning an Anſwer to the *Commercium Epiſtolicum*, by pretending that he had not ſeen it. And now he avoids it, by telling you, That the *Engliſh* ſhall not have the Pleaſure to ſee him return an Anſwer to their ſlender Reaſonings (as he calls them) and by endeavouring to engage me in Diſputes about *Philoſophy*, and about ſolving of *Problems*; both which are nothing to the Queſtion.

As for *Philoſophy*, he colludes in the Significations of Words, calling thoſe things Miracles which create no Wonder; and thoſe things occult Qualities, whoſe Cauſes are occult, tho' the Qualities themſelves be manifeſt; and thoſe things the Souls of Men, which do not animate their Bodies. His *Harmonia Præſtabilita* is miraculous, and contradicts the daily Experience of all Mankind; every Man finding in himſelf a Power of ſeeing with his Eyes, and moving his Body by his Will. He prefers Hypotheſes to Arguments of Induction drawn from Experiments, accuſes me of Opinions which are not mine; and inſtead of propoſing Queſtions to be examin'd by Experiments before they are admitted into *Philoſophy*, he propoſes Hypotheſes to be admitted and believed before they are examin'd: But all this is nothing to the *Commercium Epiſtolicum*.

He complains of the Committee of the Royal Society, as if they had acted partially in omitting what made againſt me; but he fails in proving the Accuſation. For he inſtances in a Paragraph concerning my Ignorance, pretending that they omitted it, and yet you will find it in the *Commercium Epiſtolicum*, pag. 74. lin. 10, 11. and I am not aſhamed of it. He ſaith, That he ſaw this Paragraph in the Hands of Mr. *Collins* when he was in *London* the ſecond time; that is, in *October* 1676. It is in my Letter of the 24th of *Octob.* 1676. and therefore he then ſaw that Letter. And in that and ſome other Letters writ before that Time, I deſcrib'd my Method of Fluxions. And in the ſame Letter I deſcribed alſo two general Methods of Series, one of which is now claimed from me by Mr. *Leibnitz*.

I believe you will think it reaſonable, that Mr. *Leibnitz* be conſtant to himſelf, and ſtill acknowledge what he acknowledged above 15 Years ago, and ſtill forbear to contradict what he forbore to contradict in thoſe Days.

In his Letter of the 20th of *May* 1675. he acknowledg'd the Receipt of a Letter from Mr. *Oldenburg*, dated the 15th of *April* 1675. with ſeveral converging Series contained therein. And I expect from him, that he ſtill acknowledge the Receipt thereof. Many Gentlemen of *Italy*, *France* and *Germany* (your ſelf being one of them) have ſeen the original Letters, and the Entries thereof in the old Letter-Books of the Royal Society; and the *Series* of *Gregory* is in the Letter of the 15th of *April* 1675. and in *Gregory*'s Original Letter dated the 15th of *Feb.* 1671.

In a Letter dated the 12th of *May* 1676. (ſeen by the ſame Gentlemen) he acknowledged that he then wanted the Method for finding a *Series* for the Arc whoſe Sine was given, and by conſequence that he wanted it when he wrote his Letter of the 24th of *October* 1674. and I expect that he ſtill acknowledge it.

In the *Acta Eruditorum* for *May* 1700. in Anſwer to Mr. *Fatio*, who had ſaid, That I was the oldeſt Inventor by many Years, Mr. *Leibnitz* acknowledged that no Body, ſo far as he knew, had the Method of Fluxions or Differences before me and him; and that no body before me had proved by a Specimen made publick that he had it. Here he allowed that I had the Method before it was publiſhed, or communicated by him to any Body in *Germany*; that the *Principia Philoſophiæ* were a Proof that I had it, and the firſt Specimen made publick of applying it to the difficulter Problems: And I expect that he ſtill continue to make the ſame Acknowledgment. At that Time he did not deny what Mr. *Fatio* affirmed, and nothing but want of Candor can make him unconſtant to himſelf.

In a Letter to me dated the 7th of *March* 1693. and now in the Cuſtody of the R. S. he wrote, *Mirificè amplivaveras Geometriam tuis Seriebus,* *ſed edito Principiorum opere, oſtendiſti patere tibi, quæ Analyſi receptæ non ſubſunt. Conatus ſum ego quoque Notis commodis adhibitis, quæ Differentias & Summas exhibent, Geometriam illam quam tranſcendentem appello, Analyſi quodammodo ſubjicere, nec res male proceſſit.* And what he then acknowledged, he ought ſtill to acknowledge.

In his Letter of the 21ſt of *June* 1677. writ in Anſwer to mine of the 24th of *October* 1676. wherein I deſcribed my Method partly in plain Words, and partly in Cyphers; he ſaid, That he agreed with me, that the Method of Tangents of *Sluſius* was not yet made perfect, and then ſet down a differential Method of Tangents publiſhed by Dr. *Barrow* in the Year 1670. and diſguiſed it by a new Notation, pretending that it was his own, and ſhewed how it might be improv'd, ſo as to perform thoſe Things which I had aſcribed to my Method, and concluded from thence, that mine differ'd not much from his, eſpecially ſince it facilitated Quadratures. And in the *Acta Eruditorum* for *Octob.* 1684. in publiſhing the Elements of his Method, he added, That it extended to the difficulter Problems, which without this Method, or another like it, could not be managed ſo eaſily. He underſtood therefore in thoſe Days, that in the Year 1676, when I wrote my ſaid Letter, I had a Method which did the ſame Things with the Method which he calls *Differential*, and he ought ſtill to acknowledge it; eſpecially now the Sentences in Cyphers are decyphered, and other Things in that Letter relating to the Method are fully explained, and the *Compendium* mentioned therein made publick.

In his Letter of the 27th of *Auguſt* 1676. he repreſented, that he did not believe that my Methods were ſo general as I deſcribed them in my Letter of the 13th of *June* preceeding, and affirmed that there were many Problems ſo difficult, that they did not depend upon Equations nor Quadratures, ſuch as (amongſt many others) were the Inverſe Problems of Tangents. And by theſe Words he acknowledged that he had not yet found the Reduction of Problems to differential Equations. And what he then acknowledged, he acknowledged again in the *Acta Eruditorum* for *April* 1691, *pag.* 178. and ought in Candor to acknowledge ſtill.

Doctor *Wallis* in the Preface to the two firſt Volumes of his Works, publiſhed in *April* 1695, wrote, That I in my two Letters written in the Year 1676, had explained to Mr. *Leibnitz* the Method (called by me, *The Method of Fluxions*, and by him, *The Differential Method*) invented by me Ten Years before or above (that is, in the Year 1666, or before:) and in the Letters which followed between them, Mr. *Leibnitz* had notice of this Paragraph, and did not then contradict it, nor found any Fault with it; and I expect that he ſtill forbear to contradict it.

See *Wallis*'s Vol. 3. pag. 654. lin. 12.

But as he has lately attacked me with an Accuſation which amounts to Plagiary; if he goes on to accuſe me, it lies upon him by the Laws of all Nations to prove his Accuſation, on Pain of being accounted guilty of Calumny. He hath hitherto written Letters to his Correſpondents, full of Affirmations, Complaints and Reflections, without proving any thing. But he is the Aggreſſor, and it lies upon him to prove his Charge.

I forbear to deſcend further into Particulars, you have them in the *Commercium Epiſtolicum*, and the Abſtract thereof, to both which I refer you. I am,

*SIR,*

*Your moſt Humble, and*

*moſt Obedient Servant.*

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‘ *Monſieur,*

‘ C'EST ſans doute pour l'amour de la verité que vous vous etes chargé d'une eſpece de cartel de la part de M. *Newton*. Je n'ay point voulu entrer en lice avec des enfans perdus, qu'il avoit detachés contre moy; ſoit qu'on entende celuy qui a fait l'Accuſateur ſur le fondement du *Commercium Epiſtolicum*, ſoit qu'on regarde la Preface pleine d'aigreur qu'un autre a miſe devant la nouvelle Edition de ſes Principes. Mais puiqu'il veut bien paroitre luy même, je ſeray bien aiſe de luy donner ſatisfaction.

‘ Je fus ſurpris au commencement de cette Diſpute d'apprendre qu'on m'acuſoit d'etre l'Aggreſſeur; car je ne me ſouvenois pas d'avoir parlé de M. *N.* que d'une manere fort obligeante. Mais je vis depuis qu'on abuſoit pour cela d'un Paſſage des Actes de *Leipzig* du Janvier 1705. ou il y a ces mots: *Pro differentiis L... ſianis, D. N... nus adhibet, ſemperque adhibuit Fluxiones*; ou l'Auteur des Remarques ſur le *Commercium Epiſtolicum* dit, *pag.* 108. *Senſus verborum eſt, quòd N.... nus Fluxiones differentiis L... tianis ſubſtituit.* Mais c'eſt une Interpretation maligne d'un homme qui cherchoit noiſe: Il ſemble que l'Auteur de paroles inſeré dans les Actes de *Leipzig* a voulu y obvier tout expres, par ces mots, *adhibet ſemperque adhibuit*; pour inſinuer, que ce n'eſt pas apres la veue de mes differences, mais deja auparavant, qu'il s'eſt ſervi de Fluxions. Et je defie qui que ce ſoit de donner un autre but raiſonnable à ces Paroles, *ſemper-*

‘ *que adhibuit*. Au lieu qu'on ſe ſert du mot *ſubſtituit*, en parlant de ce que le Pere-*Fabri* avoit fait apres Cavallieri. D'ou il faut conclure on que M. *N.* s'eſt laiſsé tromper par un homme qui a empoiſonné ces Paroles des Actes, qu'on ſuppoſoit n'avoir pas eté publiées ſans ma connoiſſance, & s'eſt imaginé qu'on l'accuſoit d'etre Plagiaire; ou bien qu'il a eté bien aiſe de trouver un pretexte de s'attribuer ou faire attribuer privément l'Invention du nouveau Calcul (depuis qu'il en remarquoit le ſuccés, & le bruit qu'il faiſoit dans le Monde) contre ſes Connoiſſances contraires avouées dans ſon Livre des Principes, *pag.* 253. de la premiere Edition. Si l'on avoit fait connoitre qu'on trouvoit quelque Difficulté, ou ſujet de plainte dans les Paroles des Actes de *Leipzig*, je ſuis aſſuré, que ces Meſſieurs qui ont part a ces Actes, auroient donné un plein contentement; mais il ſemble qu'on cherchoit un pretexte de rupture.

‘ Je n'ay pas eu connoiſſance du *numerous Committee of Gentlemen of ſeveral Nations relating to the Diſpute*; car on ne m'en a donné aucune part, & je ne ſay pas encore preſentement les Noms de tous ces Commiſſaires, & particulierement de ceux qui ne ſont pas des Iſles Britaniques, je ne crois pas qu'ils approuvent tout ce qui a eté mis dans l'Ouvrage publié contre moi.

‘ Il eſt aisé a croire que j'ay eté quelque temps à *Vienne*, avant que d'avoir vû le *Commercium Epiſtolicum* deja publié, quoique j'en euſſe des Nouvelles. Ainſi un ami ſachant cela, auſſi zelé pour moi que les ſeconds de M. *N.* le peuvent être pour luy, a publié une Papier, que M. *N.* appelle diffamatoire (*defamatory Letter.*) Mais cette Piece n'etant pas plus forte que ce qu'on a publié contre moi, M. *N.* n'a pas droit de s'en plaindre. Si l'on n'a pas marqué l'Auteur ni le Lieu de l'Impreſſion du Papier; on connoit aſſez le Nom & le Lieu de l'Auteur de la Lettre y inſerée d'un excellent Mathematicien que j'avois prié de dire ſon Sentiment ſur le *Commercium*, & cela ſuffit. M. *N.* (dont les Partiſans ont marqué qu'il ne leur etoit pas inconnu) l'appelle un Mathematicien ou pretendu Mathematicien; & apres avoir fait inutilement des efforts pour le gagner, il le mépriſe contre l'opinion publique, qui le met entre ceux du premier rang, & contre l'evidence des choſes verifiées par ſes découvertes.

‘ Lors que j'eus enfin le *Commercium Epiſtolicum*, je vîs qu'on s'y écartoit entirement du but, & que les Lettres qu'on publioit ne contenoient pas un mot qui peut faire revoquer en doute mon Invention du Calcul des Differences dont il s'agiſſoit. Au lieu de cela je remarquay qu'on ſe jettoit ſur les Series, ou l'on accorde l'avantage à M. *N.* & que les Remarques contenoient de Gloſes mal tournées, pour tacher de me decrier par des ſoubçons ſans fondement quelquefois ridicules, & quelquefois forgés contre la Conſcience de quelques

‘ uns de ceux qui en étoient les auteurs ou approbateurs.

‘ Pour repondre donc de point en point a l'Ouvrage publié contre moi, il falloit un autre Ouvrage auſſi grand pour le moins que celuy là, il falloit entrer dans un grand detail de quantité de Minuties paſſées il y a 30 ou 40 Ans dont je ne me ſouvenois gueres ; il me falloit chercher mes vieilles Lettres, dont pluſieurs ſe ſont perdues, outre que le plus ſouvent, je n'ay pas gardé les Minutes des miennes ; & les autres ſont enſevelies dans un grand tas de Papiers, qui je ne pouvois debrouiller qu'avec du Temps & de la Patience. Mais je n'en avois gueres le loiſir, étant chargé preſentement d'Occupations d'une toute autre Nature.

‘ De plus je remarquay que dans la Publication du *Commercium Epiſtolicum* on a ſupprimé des endroits qui pouvoient être au deſadvantage de M. *N.* au lieu qu'on n'y a rien omis de ce qu'on croyoit pouvoir tourner contre moi par des gloſes forcées. Comme je n'ay pas daigné lire le *Commercium Epiſtolicum* avec beaucoup d'attention, je me ſuis trompé dans l'Exemple que j'ay cité, n'ayant pas pris garde, ou ayant oublié qu'il s'y trouvoit ; mais j'en citéray un autre : M. *N.* avouoit dans un des ſes Lettres à M. *Collins*, qu'il ne pouvoit point venir à bout des Sections ſecondes (ou Segments ſeconds) de Spheroides ou corps ſemblables : mais on n'a point inſeré ce Paſſage ou cette Lettre dans le *Commercium Epiſtolicum* ; il auroit été plus ſincere par rapport à la Diſpute, & plus utile au public, de donner le Commerce litteraire de M. *Collins* tout entier, là ou il contenoit quelque choſe qui meritoit d'être lû ; & particulierement de ne pas tronquer les Lettres, car il y en a peu parmi mes Papiers, ou dont il me reſte des Minutes.

‘ Ainſi tout conſideré, voyant tant de marques de malignité & de chicane, je crûs indigne de moi d'entrer en diſcuſſion avec des gens qui en uſoient ſi mal. Je voyois qu'en les refutant on auroit de la peine à éviter des Reproches, & des Expreſſions fortes, telles que meritoit leur Proceedé ; & je n'avois point envie de donner ce Spectacle au Public, ayant deſſein de mieux employer mon temps, qui me doit être precieux, & mepriſant aſſez le Jugement de ceux qui ſur un tel Ouvrage voudroient prononcer contre moi, d'autant que la Societé Royale même ne la point voulu faire ; comme je l'ay appris par un Extrait de ſes Regiſtres.

‘ Je ne crois point d'avoir dit (comme M. *N.* me l'impute) que les Anglois n'auroient point le plaiſir de me voir repondre à des raiſonnements ſi minces ; car je ne crois point que tous les Anglois faſſent leur Cauſe de celle de M. *N.* il y en a de trop habiles & de trop honnêſtes pour épouſer les paſſions de quelquesuns de ſes adherens.

‘ Apres cela, il m'accuſe d'avoir voulu faire diverſion, en combattant ſa Philoſophie, & en voulant l'engager dans des Problemes; mais quant à la Philoſophie, j'ay donné publiquement quelque choſe de mes Principes ſans attaquer les ſiens; ſi ce n'eſt que par Occaſion j'en ay parlé dans des Lettres particulieres, depuis qu'on m'en a donné ſujet; & pour ce qui eſt des Problemes, je n'ay garde d'en propoſer à M. *N.* car je ne voudrois pas m'y engager quand on m'en propoſeroit à moi, nous pouvons nous en diſpencer à l'age ou nous ſommes, mais nous avons des amis qui y peuvent ſuppleer à notre Defaut.

‘ Je ne veux point entrer icy dans le detail de ce que M. *N.* dit un peu aigrement contre ma Philoſophie, car pour la ſienne, ce n'en eſt point le lieu. J'apelle *Miracle* tous Evenement qui ne peut être arrivé que par la Puiſſance du Createur, ſa Raiſon n'étant pas dans la Nature des Creatures, & quand on veut neamoins l'attribuer aux qualités ou forces des Creatures, alors j'appelle cette Qualité *une Qualité occulte à la Scholaſtique*; c'eſt à dire, qu'il eſt impoſſible de rendre manifeſte, telle que ſeroit une peſanteur primitive; car les Qualités occultes qui ne ſont point Chimeriques, ſont celles dont nous ignorons la cauſe, mais que nous n'excluons point; & j'appelle *l'Ame de l'homme* cette Subſtance ſimple qui s'apperçoit de ſe qui ſe paſſe dans le corps humain, & dont les Appetits ou Volontés ſont ſuivis par les Efforts du Corps. Je ne prefére pas les Hypotheſes aux Arguments tirez de l'Induction des Experiences, mais quelquefois on fait paſſer pour Inductions generales ce qui ne conſiſte qu'en Obſervations particulieres, & quelquefois on veut faire paſſer pour une Hypotheſe ce qui eſt demonſtratif. L'Idée que M. *N.* donne icy de mon Harmonie préetablie n'eſt pas celle qu'en ont quantité d'habiles gens hors d'Angleterre, & quelques uns en Angleterre; & je ne crois pas que vous même Monſ. en ayez eu une ſemblable, ou l'ayés maintenant, à moins que d'être bien changé.

‘ Je n'ay jamais nié qu'a mon ſecond Voyage en Angleterre j'aye vû quelques Lettres de M. *N.* chez Monſieur *Collins*, mais je n'en ay jamais vû ou M. *N.* ait expliqué ſa Methode des Fluxions, & je n'en trouve point dans le *Commercium Epiſtolicum*.

‘ Je ne pas vû non plus qu'il ait expliqué la Methode des Series que je m'attribue, je crois qu'il veut parler de celle ou je prends une Series arbitraire, je l'ay fait avant mon ſecond retour en Angleterre. Je ne nie pourtant pas, que M. *N.* n'eut pû l'avoir auſſi, & ce n'eſt pas même une Invention fort difficile.

‘ M. *N.* veut que j'avoue, & que j'accorde ce que j'ay avoué ou accordé il y a 15 Ans, ou autrement on devroit en attendre de luy autant; car il y a maintenant deux fois quinze Ans, que dans la premiere Edition de ſes Principes, *pag.* 253, 254. il m'accorde l'Invention

‘ vention du Calcul des Differences, independement de la ſienne, & depuis il s'eſt aviſé je ne ſcay comment de faire ſoutenir le contraire.

‘ Il eſt bon de ſavoir qu'a mon premier Voyage d'Angleterre en 1673. je n'avois pas la moindre connoiſſance des Series infinies, telles que M. *Mercator* venoit de donner, ny d'autres matieres de la Geometrie avancée par les dernieres Methodes, je n'etois pas même aſſez verſé dans l'Analyſe de Des Cartes, je ne traitois les Mathematiques que comme un *Parergon*, & je ne ſavois guere que la Geometrie practique vulgaire quoy que j'euſſe vû par hazard la Geometrie des indiviſibiles de *Cavalleri*, & un Livre de Pere *Leotand*, ou il donnoit les Quadratures des Lunules & Figures ſemblables, ce qui m'avois donné quelque curioſité; mais je me divertiſſois pluſtôt aux proprietez des Nombres, à quoy le petit Traité que j'avois publié preſque petit garçon de l'Art des Combinaiſons en 1666 m'avoit donné occaſion, & ayant obſervé des lors l'uſage des Differences pour les Sommes, je l'appliquay à des ſuites de Nombres. On voit bien par mes premieres Lettres échangées avec M. *Oldenbourg*, que je n'etois guere allé plus avant, auſſi n'avois je point alors la connoiſſance de M. *Collins*, quoy qu'on ait feint malicieuſement le contraire.

‘ Ce fût peu à peu que M. *Hugens* me fit entrer en ces matieres, quand je le pratiquois à *Paris*, & cela joint au Traité de M. *Mercator* (que j'avois rapporté avec moi d'Angleterre parce que M. *Pell* m'en avoit parlé) me fit trouver environ vers la fin de l'An 1673 ma Quadrature Arithmetique du Cercle qui fût fort approuvé par M. *Hugens*, & dont je parlay à M. *Oldenbourg* dans une Lettre de l'An 1674. alors ny M. *Hugens* ny moi, nous ne ſavions rien des Series de M. *N.* ny de M. *Gregory*. Ainſi je crus être le premier qui eut donné la valeur du Cercle par une ſuite de Nombres rationaux; & M. *Hugens* le crut auſſi, j'en écrivis ſur ce ton là à M. *Oldenbourg* qui me repondit qu'on avoit deja de telles Series en Angleterre, & l'on voit par ma Lettre du 15 Juillet de 1674. & par la reponſe de M. *Oldenbourg* du 8 Decembre de la même Année que je n'en devois avoir aucune connoiſſance alors, autrement M. *Oldenbourg* n'auroit pas manqué de me le faire ſentir, ſi luy ou M. *Gollins* m'en euſſent communiqué quelque choſe; mais je ne ſavois pas alors les Extractions des Racines, des Equations par des Series, ny les Regreſſions ou l'Extraction d'une Equation infinie; j'etois encore un peu neuf en ces matieres, mais je trouvai pourtant bientôt ma Methode generale par des Series arbitraires; & j'entray enfin dans mon Calcul des differences, ou les obſervations que j'avois faites encore fort jeune ſur les differences des ſuites des Nombres, contribuerent à m'ouvrir les yeux; car ce n'eſt pas par les Fluxions des lignes, mais par les differences

‘ ferences des Nombres que j'y ſuis venu, en conſiderant enfin que ces differences appliquées aux grandeurs qui croiſſent continuellement, s'evanouiſſent en comparaiſon des grandeurs differentes,au lieu qu'elles ſubſiſtent dans les ſuites des Nombres. Et je crois que cette voye eſt la plus analytique, le Calcul Geometrique des differences qui eſt le même que celuy des Fluxions, n'etant qu'un cas ſpecial, devient plus commode par les evanouiſſements.

‘ M. *N.* allegue par apres les paſſages, ou j'accorde qu'il y a un Calcul approchant de mon Calcul des differences, mais il pourra bien ſe ſouvenir qu'il m'en a accordé autant,& s'il luy eſt permis de ſe retracter, pourquoy ne me ſera t'il pas permis d'en faire autant ? Sur tout apres les veriſimilitudes que M. *Bernouilli* a remarquées, j'ay une ſi grande opinion de la Candeur de M. *N.* que je l'ay crû ſur ſa parole, mais le voyant conniver à des accuſations dont la fauſſeté luy eſt connue, il etoit naturel que je commençaſſe de douter.

‘ Je ne puis avouer ny deſavouer aujourdhuy d'avoir écrit, ou receu des Lettres écrites il y a plus de 40 ans telles qu'on les a publiées, je ſuis obligé de m'en rapporter à ce qui ſe trouve dans les Papiers qu'on cite, mais je ne remarque rien contre moi dans celles que M. *N.* allegue du 15 Avril & 20 May 1675. & du 24 Octobre 1676. ſinon dans les fauſſetez du Gloſateur, je crois que c'etoit purement par diſtraction dans un ſejour comme celuy de *Paris*, ou je m'occupois à bien d'autres choſes encore qu'aux Mathematiques, & par l'éloignement que j'avois des Calculs, dont je craignois la longueur, que j'ay demandé quelquefois à M. *Oldenbourg* la Demonſtration ou la Methode d'arriver a certaines choſes ou j'aurois bien pû arriver moi même. Par exemple, je crois d'avoir deja eu au douze de May 1676. ma Methode d'une Series Arbitraire, qui m'auroit pû mener a des Series dont j'y demande la raiſon. Car ayant conſulté mon vieux Traité de la Quadrature Arithmetique achevé quelque temps avant ma ſortie de *France*, je me ſers de la Series Arbitraire, cependant les Series marquées dans cette Lettre ſont une choſe dont je conſens dêtre redevable à d'autres, & je crois de ne les avoir pas même connues en 1674.

‘ N'entendant pas bien ce que M. *N.* allegue des Actes de *Leipzig* de May 1700, j'y ay regardé, & je trouve qu'il n'en a pas bien pris le ſens. Il n'y eſt point parlé de l'Invention du nouveau Calcul des differences, mais d'un artifice particulier des *Maximis & Minimis*, qui en eſt independant, & dont je m'etois aviſé bien du temps avant que M. *Bernouilli* eut propoſé ſon Probleme de la plus courte deſcente, mais dont je jugeois que M. *N.* ſe devoit être aviſé auſſi, lors qu'il avoit donné la figure de ſon Vaiſſeau dans ſes Principes. Ainſi j'ay voulu dire, qu'il a fait connoître publiquement avant moi, qu'il poſ-

‘ ſedoit cet Artifice, ce que je ne pouvois pas dire du Calcul des Differences & des Fluxions puiſque j'en avois fait voir l'utilité publiquement avant la publication de ce Livre. Cet Artifice particulier de *Maximis & Minimis* n'eſt point neceſſaire, quand ils s'agit ſimplement d'une grandeur (car alors la Methode de M. *Fermat* perfectionée par les nouveaux Calculs ſuffit) mais quand il s'agit de toute une Figure qui doit faire le mieux un effect demandé, il faut autre choſe.

‘ M. *N.* hazarde icy une accuſation mais, qui va tomber ſur luy même. Il pretend que ce que j'ay écrit pour luy à M. *Oldenbourg* en 1677. eſt un deguiſement de la Methode de M. *Barrow*. Mais comme M. *N.* avoue dans la *Pag.* 253 & 254. de la premiere Edition de ſes Principes, *Me ipſi (tunc Methodum communicâſſe à Methodo ipſius vix abludentem præterquam in verborum & notarum formulis*, il s'enſuivra que ſa Methode auſſi n'eſt qu'un déguiſement de celle de M. *Barrow*.

‘ Je croy que luy & moi nous ſerons aiſement quites de cette accuſation : Car une infinité de gens liront le Livre de M. *Barrow*, ſans y trouver nôtre Calcul ; il eſt vray que feu M. *Tſchirnhaus* qui s'apperceut un peu tard de l'avantage de ce Calcul pretendoit qu'on pouvoit arriver à tout cela par les Methodes de M. *Barrow*. Comme *l'Abbé Catelan* François pretendit que même l'Analyſe de Des Cartes ſuffiſoit pour toute ces choſes, mais il etoit plus aiſé de le dire que de le montrer.

‘ Cependant ſi quelqu'un a profité de M. *Barrow*, ce ſera plûs tôt M. *N.* qui a étudié ſous luy que moi qui (autant que je puis m'en ſouvenir,) n'ay veu les Livres de M. *Barrow* qu'à mon ſecond Voyage d'Angleterre, & ne les ay jamais lûs avec attention, parce qu'en voyant le Livre je m'apperçus que par la conſideration du Triangle Characteriſtique (dont les Cotez ſon les Elements de l'Abſciſſe, de l'Ordonnée & de la Courbe) ſemblable a quelque Triangle aſſignable, j'etois venu comme en me jouant aux Quadratures, Surfaces & Solides dont M. *Barrow* avoit remply un Chapitre des plus conſiderables de ſes Leçons, outre que je ne ſuis venu a mon Calcul des Differences dans la Geometrie qu'apres en avoir vû l'uſage (mais moins conſiderable) dans les Nombres, comme mes premieres Lettres dans le *Commercium Epiſtolicum* le peuvent inſinuer. Il ſe peut que M. *Barrow* en ait plus ſçu qu'il n'a pas dit dans ſon Livre, & qu'il a donné des Lumieres à M. *N.* que nous ne ſavons pas, & ſi j'etois ſemblable a certains temeraires, je pourrois aſſeurer ſur de ſimples ſoubçons ſans autre fondement que le Calcul des Fluxions de M. *N.* qu'el qu'il puiſſe être, luy a eté enſeigné par M. *Barrow*.

‘ On peut bien juger que lors que j'ay parlé en *1676.* des Problemes qui ne dependoient, ny des Equations, ny des Quadratures, j'ay voulu parler des Equations telles qu'on connoiſſoit alors dans le monde; c'eſt à dire, des Equations de l'Analyſe ordinaire. Et on le peut juger de ce que j'ajoute les Quadratures comme quelque choſe de plus que ces Equations. Mais les Equations Differentielles vont au de là même des Quadratures, & l'on voit bien que j'entendois même parler des Problemes qui vont a ces ſortes d'Equations inconnues alors au Public; cette objection ſe trouvoit deja dans les remarques an *Commercium*, mais je n'avois point crû que M. *N.* etoit capable de l'employer.

‘ Je juge par un endroit de ma Lettre du 27 d'Aouſt *1676.* (*pag*. 65. du *Commercium Epiſtolicum*) que je devois deja avoir alors l'ouverture du Calcul des Differences; car j'ay dit d'avoir reſolu d'abord par une certaine Analyſe (*certa Analyſi ſolvi*) le Probleme de M. de *Beaune* proposé à M. Des Cartes; ſi cette Analyſe n'etoit que cela, ou le peut reſoudre ſans cela; & je crois que Monſieur *Hugens* & Monſieur *Barrow* l'auroient donné au beſoin comme beaucoup d'autres choſes, mais ſelon ma maniere de noter, ce n'eſt qu'un jeu; je trouve une petite faute dans cette Page, il y a *ludus naturæ* au lieu de ~~*luſus*~~ *naturæ*, mais cette faute etoit ancienne, & ſe devoit deja trouver dans la copie de ma Lettre du 24 d'Octobre *1676. pag*. 86. du *Commercium*.) *Hos caſus vix numeraverim inter luſus naturæ.* Je n'avois point entendu ce qu'il vouloit dire, mais à preſent je vois l'origine de la mépriſe.

hujus

‘ Je ne ſaurois dire aujourdhuy ſi j'ay remarqué le paſſage de M. *Wallis*, ou il dit que M. *N.* ſavoit deja la Methode des Fluxions en *1666.* Mais quand je l'aurois remarqué, je l'aurois laiſsé paſſer apparemment, etant fort porté alors à croire M. *N.* ſur ſa parole. Mais ſon dernier procedé m'a forcé d'être plus circomſpect à cet égard.

‘ M. *N.* dit que je l'ay accusé d'être plagiaire, mais ou eſt ce que je l'ay fait? Ce ſont ſes adherens qui ont paru intenter cette accuſation contre moi, & il y a connivé. Je ne ſay pas s'il adopte entierement ce qu'ils ont publié, mais je conviens avec luy, que la malice de celuy qui intente une telle accuſation ſans la prouver, le rend coupable de calomnie.

‘ Il finit ſa Lettre en m'accuſant dêtre l'aggreſſeur, & j'ay commencé celle cy en prouvant le contraire. Il ſera fort aisé de vuider ce point preliminaire. Il y a eu du meſentendu, mais ce n'eſt pas ma faute; au reſte je ſuis avec Zele.

*Monſieur,*

*Hannover,* *Vôtre tres Humble & tres Obeiſſant Serviteur.*

ce 9 d'Avril 1716. Leibnitz.

*Cum* D. Leibnitius *adduci non poſſet, ut vel Commercio Epiſtolico reſponderet, vel probaret quæ pro habitu affirmabat, cumque præcedentes Epiſtolas in Galliam prius mitteret quam earum tertia in Angliam veniret, & prætenderet ſe hoc facere, ut teſtes haberet, & alias etiam adhiberet contumelias*: Newtonus *minime reſcripſit, ſed Obſervationes ſequentes in Epiſtolam illam tertiam ſcriptas, cum amicis ſolummodo communicavit.*

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## OBSERVATIONS upon the preceding EPISTLE.

MR. *Leibnitz* by his Letter of the 29th of *December* 1711. juſtified the Paſſage in the *Acta Eruditorum* for *January* 1705. *pag.* 34, and 35. and thereby made it his own, and now endeavours in vain to excuſe it, pretending that the Words *adhibet ſemperque adhibuit* are maliciouſly interpreted by the Word *ſubſtituit*. But in the Interpretation which he would put upon the Place, he omits the Words *igitur* and *quemadmodum*, the firſt of which makes the Words, *ſemperque adhibuit*, a Conſequence of what went before, and the latter makes them equipolent to *ſubſtituit*; neither of which can be true in the Senſe which Mr. *Leibnitz* endeavours now to put upon the Words. He has therefore accuſed me. In both his Letters to Dr. *Sloan*, (that dated the 4th of *March*, and that dated the 29th of *December* 1711.) he preſſed the Royal Society to condemn Dr. *Keill*; and before I meddled in this matter challenged me to declare my Opinion. His Words in his ſecond Letter are; *Itaque veſtræ æquitati committo, annon coercendæ ſint vanæ & injuſtæ* [Keillii] *vociferationes, quas ipſi* Newtonio *viro inſigni, & geſtorum optimè conſcio, improbari arbitror; ejuſque ſententiæ ſuæ libenter daturum indicia mihi perſuadeo.* The Words are civil, but the Senſe is, That I muſt either condemn Dr. *Keill*, or enter into a Quarrel with Mr. *Leibnitz*, as has happen'd; and therefore he is the Aggreſſor. For it is very well known here, that I conſtantly endeavoured to avoid theſe Diſputes, till they were preſſed upon the Royal Society and me.

In his Letter of the 4th of *March ſt. n.* 1711. he preſſed the Royal Society to condemn Dr. *Keill* without hearing both Parties; and when the Doctor put in his Anſwer, Mr. *Leibnitz* refuſed to give his Reaſons againſt the Doctor, and call'd it *Injuſtice* to expect it from him, and yet perſiſted in preſſing them againſt him, and thereby put them upon a Neceſſity of appointing a Committee to ſearch out old Papers, and give their Opinion upon them. If they did it without him, it was his own Fault: He was for over ruling them, and called it *Injuſtice* to expect that he ſhould defend his Candor, and plead before them. If they gave him no Opportunity to except againſt any of the Committee, it was becauſe he refuſed to be heard, and they had a ſufficient Authority to appoint a Committee without him, and he had no Right to except againſt what they did for their own Satisfaction. If they have not yet given Judgment againſt him, it is becauſe the Committee did not act as a Jury, nor the Royal Society as a formal Court of Juſtice. The Committee examined old Letters and Papers, and gave their Opinion upon them alone, and left room for Mr. *Leibnitz* to produce further Evidence for himſelf. And it is ſufficient that the Society ordered their Report, with the Papers upon which it was grounded, to be publiſhed; and that Mr. *Leibnitz* in all the three Years and four Months which are ſince elapſed, has not been able to produce any further Proof againſt Dr. *Keill* than what was then before them.

Mr. *Leibnitz* ſaith, That the Letter which I call *Defamatory*, being no ſharper than that which has been publiſhed againſt him, I have no Reaſon to complain. But the ſharpneſs of the Letter lies in Accuſations and Reflections, without any Proof; which way of Writing is unlawful and infamous, and never uſed but in a bad Cauſe. The ſharpneſs of the *Commercium* lies in Facts which are lawful and fit to be produced. The Letter was publiſhed in a clandeſtine, back-biting manner (as defamatory Papers uſe to be) without the Name of the Author, or Mathematician, or Printer, or City where it was printed, and was diſperſed above two Years before we were told that the Mathematician was *John Bernouilli*; the *Commercium* was printed openly at *London* by Order of the Royal Society.

The Mathematician to whom Mr. *Leibnitz* appealed from the Royal Society, I called a Mathematician, or pretended Mathematician, not to diſparage the Skill of Mr. *Bernouilli*, but becauſe the Mathematician in his Letter of the 7th of *June* 1673, cited Mr. *Bernouilli* as a Perſon diſtinct from himſelf; and Mr. *Leibnitz* lately cauſed that Letter to be reprinted without the Citation, and tells us, that the Mathematician was Mr. *Bernouilli* himſelf: And whether the Mathematician, or Mr. *Leibnitz* is to be believed, I do not know. Mr. *Bernouilli* had the differential Method from M. *Leibnitz*, and was the chief of his Diſciples, and gave his Opinion for his Maſter in the *Acta Leipſica* before he ſaw the *Commercium Epiſtolicum*; at which Time he was *homo novus, & rerum anteactarum parum peritus*, as Mr. *Leibnitz* objected againſt Dr. *Keill*; and what he wrote after he ſaw the *Commercium* was in his own Defence, and his Skill in Mathematicks will not mend the matter.

He complains that the Committee have gone out of the way, in falling upon the Method of *Series*: But he ſhould conſider that both Methods are but two Branches of one general Method of *Series*: I joyn'd them together in my Analyſis I interwove them in the Tract which I wrote in the Year 1671, as I ſaid in my Letters of the 10th of *December* 1672, and the 24th of *October* 1676. In my Letter of the 13th of *June* 1676, I ſaid, that my Method of *Series* extended to almoſt all Problems, but became not general without ſome other Methods, meaning (as I ſaid in my next Letter) the Method of Fluxions, and the Method of Arbitrary *Series*; and now to take thoſe other Methods from me, is to reſtrain and ſtint the Method of *Series*, and make it ceaſe to be general. In my Letter of the 24th of *October* 1676, I called all theſe Methods together, my general Method. See the *Commercium Epiſtolicum, pag.* 86. *lin.* 16. And if Mr. *Leibnitz* has been tearing this general Method in pieces, and taking from me firſt one Part, and then another Part, whereby the reſt is maimed, he has given a juſt Occaſion to the Committee to conſider the Whole. It is alſo to be obſerved, That he is perpetually giving Teſtimony for himſelf, and it's allowed in all Courts of Juſtice to ſpeak to the Credit of the Witneſs.

He repreſents, That the Committee of the Royal Society have omitted Things which made againſt me, and printed every Thing which could be turned againſt him by ſtrained Gloſſes; and to make this appear, he produces in his laſt Letter but one, an Inſtance of my Ignorance omitted by them, but confeſſes now that he was miſtaken in ſaying that it was omitted, and ſaith that he will cite another Inſtance. He ſaith, That in one of my Letters to Mr. *Collins*, I owned that I could not find the ſecond Segments of Sphæroids, and that the Committee have omitted this. If they had omitted ſuch a Paſſage, I think they would have done right, it being nothing to the purpoſe. But on the contrary, Mr. *Collins* in a Letter to Mr. *James Gregory* the 24th of *Decemb.* 1670, and in another to Mr. *Bertet* the 21ſt of *Feb.* 1671, both printed in the *Commercium Epiſtolicum, pag.* 24, 26. wrote, That my Method extended to ſecond Segments of round Solids. And Mr. *Oldenbourg* wrote the ſame thing to Mr. *Leibnitz* himſelf the 8th of *December* 1674. See the *Commercium Epiſtolicum, pag.* 39. So you ſee that Mr. *Leibnitz* hath accuſed the Committee of the Royal Society, without knowing the Truth of his Accuſation, and therefore is guilty of a Miſdemeanour. The Committee were ſo far from acting corruptly againſt Mr. *Leibnitz*, that they took no Notice of his Ignorance of Geometry in thoſe Days, and omitted ſeveral other things which made ſtrongly againſt him, ſuch as were the two Letters in my Cuſtody, and the Paragraph in the Preface to the two firſt Volumes of Dr. *Wallis*'s Works relating to this matter, and that a Copy of *Gregory*'s Letter of the 5th of *Septemb.* 1670, was ſent to Mr. *Leibnitz* in *June* 1676, amongſt the Extracts of *Gregory*'s Letters.

The Committee in their Report, affirmed, That they had extracted from the ancient Letters, Letter-Books, and Papers, what related to the Matter referred to them: All which Extracts delivered by them to the Society, they believed to be genuine and authentick. Mr. *Leibnitz* accuſes them for not printing the Letters entire (including as well what did not relate to the matter referred to them, as what did relate to it,) as if it were not lawful to cite a Paragraph out of a Book, without citing the whole Book. Thus he complains, that the *Commercium Epiſtolicum* ſhould have been much bigger. But when he is to anſwer it, he complains that it is too big, and would require an Anſwer as big as it ſelf. And ſo the ancient Letters and Papers muſt be laid aſide, and the Queſtion muſt be run off into a Squabble about Philoſophy and other matters: And the great Mathematician, who in his Letter to Mr. *Leibnitz*, dated rhe 7th of *June* 1713, concealed his Name, that he might paſs for an impartial Judge, muſt now pull off his Mask, and become a Party-man in this Squabble, and ſend a Challenge by Mr. *Leibnitz* to the Mathematicians in *England*, as if a Duel, or perhaps a Battel with his Army of Diſciples, were a fitter way to decide the Truth, than an Appeal to ancient and authentick Writings; and Mathematicks muſt henceforward be filled with Atchievements in Knight-errantry, inſtead of Reaſons and Demonſtrations.

Mr. *Leibnitz* acknowledges, that when he was in *London* the ſecond time, he ſaw ſome of my Letters in the Hands of Mr. *Collins*, eſpecially thoſe relating to *Series*; and he has named two of them which he then ſaw, *viz.* that dated the 24th of *October* 1676, and that in which he pretends that I confeſſed my Ignorance of ſecond Segments. And no doubt he would principally deſire to ſee the Letter which contained the chief of my *Series*, and particularly that which contained thoſe two for finding the Arc by the Sine, and the Sine by the Arc, with the Demonſtration thereof, which a few Months before he had deſired Mr. *Oldenbourg* to procure from Mr. *Collins*; that is, the *Analyſis per æquationes numero terminorum infinitas.* But yet he tells us, that he never ſaw where I explained my Method of Fluxions, and that he finds nothing of it in the *Commercium Epiſtolicum*, where that Analyſis and my Letters of the 10th of *December* 1672, 13th of *June* 1676, and the 24th of *Octob.* 1676, are publiſhed. I ſuppoſe he means, becauſe he finds no prick'd Letters there. And by the ſame way of arguing, he and Mr. *Bernouilli* may pretend that they find nothing of the Method of Fluxions in the Introduction to the Book of Quadratures.

He ſaith alſo, That he never ſaw where I explain the Method claimed by me, in which he aſſumes an arbitrary *Series*. If he pleaſes to look into the *Commercium Epiſtolicum*, *pag.* 56 and 86, he will there ſee that I had that Method in the Year 1676, and five Years before. And

Dr. *Wallis* in the ſecond Volume of his Works, *pag.* 393. *lin.* 32. has told him, That this Method needs no further Explication than what I there gave of it. Mr. *Leibnitz* might find it himſelf, but not ſo early; and ſecond Inventors have no Right.

He pretends, that in my Book of Principles, *pag.* 253, 254, I allowed him the Invention of the *Calculus Differentialis*, independenrly of my own; and that to attribute this Invention to my ſelf, is contrary to my Knowledge there avowed. But in the Paragraph there refered unto, I do not find one Word to this purpoſe. On the contrary, I there repreſent, that I ſent Notice of my Method to Mr. *Leibnitz* before he ſent Notice of his Method to me; and left him to make it appear that he had found his Method before the Date of my Letter; that is, eight Months at the leaſt before the Date of his own. And by referring to the Letters which paſſed between Mr. *Leibnitz* and me ten Years before, I left the Reader to conſult thoſe Letters, and interpret the Paragraph thereby. For by thoſe Letters he would ſee that I wrote a Tract on that Method, and the Method of *Series* together, five Years before the writing of thoſe Letters; that is, in the Year 1671. And theſe Hints were as much as was proper in that ſhort Paragraph, it being beſides the Deſign of that Book to enter into Diſputes about theſe Matters.

He ſaith, That when he was in *London* the firſt Time, which was in *January* and *February* 1673, he knew nothing of infinite *Series*, nor of the advanced Geometry, nor was then acquainted with Mr. *Collins*, as ſome have maliciouſly feigned. But who hath feigned this, or what need there was of feigning it, I do not know. At that Time Dr. *Pell* gave him Notice of *Mercator*'s *Series* for the *Hyperbola*, and he carried *Mercator*'s Book with him to *Paris*, tho' he did not yet underſtand the higher Geometry. And any of thoſe to whom Mr. *Collins* had communicated mine and *Gregory*'s *Series*, might give him Notice of them, without his being acquainted with Mr. *Collins*.

He ſaith, That after his coming from *London* to *Paris*, his firſt Letters were of other matters than Geometrical, till Mr. *Huygens* had inſtructed him in theſe Things; and that he found the Arithmetical Quadrature of the Circle towards the End of the Year 1683, and began to write of it to Mr. *Oldenbourg* the next Year, and found the general Method by Arbitrary *Series* a little after, and the *Differential Calculus* in the Year 1676, deducing it from the *Series* of Numbers; and that in his Letter of the 27th of *Auguſt* 1676, by the Words, *certa Analyſi*, he meant the Differential Analyſis. And am not I as good a Witneſs that I invented the Methods of *Series* and Fluxions in the Year 1665, and improved them in the Year 1666, and that I ſtill have in my Cuſtody ſeveral Mathematical Papers written in the Years

1664.

1664, 1665, and 1666, ſome of which happen to be dated; and that in one of them dated the 13th of *Novemb.* 1665, the direct Method of Fluxions is ſet down in theſe Words:

PROB. *An Equation being given, expreſſing the Relation of two or more Lines, $x$, $y$, $z$, &c. deſcribed in the ſame time by two or more moving Bodies, A, B, C, &c. to find the Relation of their Velocities, $p$, $q$, $r$, &c.*

Reſolution. *Set all the Terms on one ſide of the Equation, that they become equal to nothing. Multiply each Term by ſo many times $\frac{p}{x}$ as $x$ hath Dimenſions in that Term.* Secondly, *Multiply each Term by ſo many Times $\frac{q}{y}$ as $y$ hath Dimenſions in it.* Thirdly, *Multiply each Term by ſo many Times $\frac{r}{z}$ as $z$ hath Dimenſions in it,* &c. *The Sum of all theſe Products ſhall be equal to nothing. Which Equation gives the Relation of $p$, $q$, $r$,* &c. And that this Reſolution is there illuſtrated with Examples, and demonſtrated, and applied to Problems about Tangents, and the Curvature of Curves. And that in another Paper dated the 16th of *May* 1666, a general Method of reſolving Problems by Motion, is ſet down in Seven Propoſitions, the laſt of which is the ſame with the Problem contained in the aforeſaid Paper of the 13th of *Novemb.* 1665. And that in a ſmall Tract written in *Novemb.* 1666. the ſame Seven Propoſitions are ſet down again, and the Seventh is improved by ſhewing how to proceed without ſticking at Fractions or Surds, or ſuch Quantities as are now called *Tranſcendent*. And that an Eighth Propoſition is here added, containing the Inverſe Method of Fluxions ſo far as I had then attained it, namely, by Quadratures of Curvilinear Figures, and particularly by the three Rules upon which the *Analyſis per Æquationes numero terminorum infinitas*, is founded, and by moſt of the Theorems ſet down in the *Scholium* to the Tenth Propoſition of the Book of Quadratures. And that in this Tract, when the Area ariſing from any of the Terms in the Valor of the Ordinate cannot be expreſſed by Vulgar Analyſis, I repreſent it by prefixing the Symbol $\Box$ to the Term. As if the *Abſciſſa* be $x$, and the Ordinate $ax - b + \frac{bb}{a+x}$, the Area will be $\frac{1}{2}axx - bx + \Box\frac{bb}{a+x}$. And that in the ſame Tract I ſometimes uſed a Letter with one Prick for Quantities involving firſt Fluxions; and the ſame Letter with two Pricks for Quantities involving ſecond Fluxions. And that a larger Tract which I wrote in the Year 1671, and mentioned in my Letter of the 24th of *Octob.* 1676, was founded upon this ſmaller Tract, and began with the Reduction of finite Quantities to converging *Series*; and with the Solution of theſe two Problems: 1. *Relatione Quantitatum fluentium inter ſe data, Fluxionum relationem determinare.* 2. *Expoſita æquatione Fluxiones*

*Quantitatum involvente, invenire relationem Quantitatum inter ſe.* And that when I wrote this Tract, I had made my Analyſis compoſed of the Methods of *Series* and Fluxions together, ſo univerſal, as to reach to almoſt all Sorts of Problems, as I mentioned in my Letter of the 13th of *June* 1676. and that this is the Method deſcribed in my Letter of the 10th of *Decemb.* 1672.

In the Year 1684. Mr. *Leibnitz* publiſhed only the Elements of the *Calculus Differentialis*, and applied them to Queſtions about Tangents, and *Maxima & Minima*, as *Fermat* and *Gregory* had done before; and ſhewed how to proceed in theſe Queſtions, without taking away Surds, but proceeded not to the higher Problems. The *Principia Mathematica* gave the firſt Inſtances made publick of applying this *Calculus* to the higher Problems; and I underſtood Mr. *Leibnitz* in this Senſe in what I ſaid concerning the *Acta Eruditorum* for *May* 1700. *pag.* 206. But Mr. *Leibnitz* obſerves, that what was there ſaid by him, relates only to a particular Artifice *de Maximis & Minimis*, with which he there allowed that I was acquainted, when I gave the Figure of my Veſſel in my Principles. But this Artifice depending upon the Differential Method as an Improvement thereof, and being the Artifice by which they ſolved the Problems which they value themſelves moſt upon (thoſe of the *Linea celerrimi Deſcenſus*, and the *Linea Catenaria* and *Velaria*,) and which Mr. *Leibnitz* there calls *a Method of the Higheſt Moment, and greateſt Extent*; I content my ſelf with his Acknowledgment, that I was the firſt who proved by a Specimen made publick, that I had this Artifice.

In the Year 1689. Mr. *Leibnitz* publiſhed the principal Propoſitions of this Book as his own, in three Papers, called, *Epiſtola de Lineis opticis, Schediaſma de reſiſtentiâ Medii & Motu projectilium gravium in medio reſiſtente, & Tentamen de Motuum Cœleſtium Cauſis*; pretending that he had found them all before that Book came abroad. And to make the principal Propoſition his own, he adapted to it an erroneous Demonſtration, and thereby diſcovered, that he did not yet underſtand how to work in ſecond Differences. And this was the ſecond Specimen made publick, of applying the Method to the higher Problems. Hitherto this Method made no Noiſe, but within a Year or two began to be celebrated.

Dr. *Barrow* printed his Differential Merhod of Tangents in the Year 1670. Mr. *Gregory* from this Method, compared with his own, deduced a general Method of Tangents without Calculation; and by his Letter of the 5th of *Septemb.* 1670, gave Notice thereof to Mr. *Collins*. *Sluſius*, in *Novemb.* 1672. gave Notice of the like Method to Mr. *Oldenbourg*. In my Letter of the 10th of *Decemb.* 1672, I ſent the like Method to Mr. *Collins*, and added, That I mentioned it to Dr. *Barrow*

when

when he was printing his Lectures; and that I took the Methods of *Gregory* and *Sluſius* to be the ſame with mine, and that it was but a Branch or Corollary of a general Method, which without any troubleſome Calculation extended not only to Tangents, but alſo to other abſtruſer Sorts of Problems concerning the Crookedneſs, Area's, Lengths, Centers of Gravity of Curves, *&c.* and did all this, even without freeing Equations from Surds; and I added, That I had interwoven this Method with that of Infinite *Series*, meaning, in the Tract which I wrote in the Year 1671. Copies of theſe Two Letters were ſent to Mr. *Leibnitz* by Mr. *Oldenburg* in the Extracts of *Gregory*'s Letters, in *June* 1676; and Mr. *Leibnitz* in his Letter of the 21ſt of *June* 1677, ſent nothing more back than what he had Notice of by theſe two Letters, namely, Dr. *Barrow*'s *Differential Method of Tangents* diſguiſed by a new Notation, and extended to the Method of Tangents of *Gregory* and *Sluſius*, and to Equations involving Surds, and to Quadratures. But this is not the Caſe between me and Dr. *Barrow*. He ſaw my Tract of Analyſis in the Year 1699, and was pleaſed with it. And before his Lectures came abroad, I had deduced the Method of Tangents of *Gregory* and *Sluſius* from my general Method. But Mr. *Leibnitz* in thoſe Days knew nothing of the higher Geometty, nor was yet acquainted with the vulgar Algebra.

In his Letter of the 27th of *Auguſt* 1676, he wrote thus: *Quod dicere videmini pleraſque difficultates (exceptis Problematibus Diophantæis) ad Series infinitas reduci, id mihi non videtur. Sunt enim multa uſque adeo mira & implexa, ut neque ab æquationibus pendeant, neque ex Quadraturis. Qualia ſunt ex multis aliis Problemata Methodi tangentium inverſæ.* And when I anſwered, That ſuch Problems were in my Power, he replied (in his Letter of the 21ſt of *June* 1676,) That he conceived that I meant by *Infinite Series*, but he meant by *Vulgar Equations*. See the Anſwer to this in the *Commercium Epiſtolicum*, *pag.* 92.

He ſaith, That one may judge, that when he wrote his Letter of the 27th of *Auguſt* 1676, he had ſome Entrance into the Differential *Calculus*, becauſe he ſaid there, that he had ſolved the Problem of *Beaune certa Analyſi*, by a certain Analyſis. But what if that Problem may be ſolved *certa Analyſi* without the Differential Method? For no further Analyſis is requiſite than this; That the Ordinate of the Curve deſired, increaſes or decreaſes in Geometrical Progreſſion, when the *Abſciſſa* increaſes in Arithmetical, and therefore the *Abſciſſa* and Ordinate have the ſame Relation to one another, as the Logarithm and its Number. And to infer from this, that Mr. *Leibnitz* had Entrance into the Differential Method; is as if one ſhould ſay, That *Archimedes* had Entrance into it, becauſe he drew Tangents to the Spiral, ſquared the *Parabola*, and found the Proportion between the Sphere and the

Cilynder, or that *Cavallerius*, *Fermat* and *Wallis* had Entrance into it, becauſe they did many more things of this kind.

### *P. S.*

WHEN the Committee of the Royal Society publiſhed the *Commercium Epiſtolicum*, the Letters and Papers in my Cuſtody were not produced. Among them were the following Letter of Mr. *Leibnitz*, dated $\frac{7}{17}$ of *March* 1693, and a Letter of Dr. *Wallis*'s, dated the 10th of *April* 1695; both which upon a freſh Occaſion two Years ago, were produced, examined, and left in the Archives of the Royal Society. The firſt ſhews what Opinion Mr. *Leibnitz* had of this matter before he knew my Symbols, or any thing more of the Method of Fluxions than what he learnt from my Letters and Papers writ in or before the Year 1676, or from the *Principia Philoſophiæ Mathematica*, and by Conſequence before I could deceive him; and that he then gave me the Precedence. The ſecond (compar'd with the Preface to the Doctor's Works) ſhews what Opinion the *Engliſh* Mathematicians, and ſome others abroad, had of this Matter, when they heard that the Differential Method began to be celebrated in *Holland* as invented by Mr. *Leibnitz*. The firſt of theſe two Letters, and Part of the ſecond, are hereunto ſubjoyn'd.

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ILLUSTRI VIRO

## ISAACO NEWTONO

*Godefridus Gulielmus Leibnitius*, S. P. D.

'QUantum Tibi Scientiam rerum Mathematicarum totiuſque Naturæ debere arbitrer, occaſione data, etiam publice ſum profeſſus. Mirifice ampliavaveras Geometriam tuis Seriebus, ſed edito Principiorum opere oſtendiſti patere Tibi, etiam quæ Analyſi receptæ non ſubſunt. Conatus ſum ego quoque Notis commodis adhibitis, quæ differentias & Summas exhibent, Geometriam illam quam Tranſcendentem appello, Analyſi quodammodo ſubjicere, nec res male proceſſit. Sed à Te adhuc magni aliquod expecto ad Summam manum imponendam, tum ut Problemata, quæ ex data Tangentium proprietate quærunt Lineas, reducantur optime ad Quadraturas; tum ut Quadraturæ ipſæ (quod valde vellem,) reducantur ad Curvarum rectificationes, ubique ſuperficierum aut corporum dimenſionibus ſimpliciores.

‘ Sed ſuper omnia optem, ut Geometricis abſolutis, naturam, uti ‘ cœpiſti Mathematice tractare pergas; in quo genere certe tu unus ‘ cum pauciſſimis ingens opere pretium feciſti. Mirificum eſt quod ‘ inveniſti Ellipſes Keplerianas prodire, ſi tantummodo attractio, ſive ‘ gravitatio, & trajectio in Planeta concipiantur. Tametſi enim eò ‘ inclinem, ut credam hæc omnia fluidi ambientis motu ſive effici, ſive ‘ regi, analogia gravitatis & magnetiſmi apud nos, nihil tamen eas ‘ res dignitati & veritati inventi tui detraxerit.

‘ Quæ ſummus & ipſe Mathematicus Chriſtianus Huygenius in tua ‘ notavit, Appendice Libelli de Cauſa Luminis & Gravitatis, expenſa ‘ Tibi non dubito; & ſententiam viciſſim tuam velim. Veſtra enim ‘ amica collatione potiſſimum, qui in hoc genere eminetis, erui veritas ‘ poteſt.

‘ Cum vero maximum tu quoque lumen ipſi Dioptricæ intuleris, ‘ explicatis colorum Phænomenis inexpectatis, velim quid ſentias de ‘ Huygeniana explicatione radiationis, utique ingenioſiſſima, cùm feliciter ‘ adeo prodeat lex ſinuum. Significavit mihi Huygenius, neſcio quæ ‘ nova Phænomena colorum ſibi à Te communicata. Ego valde optem ‘ ut ratio colorum, quos fixos vocant, ex apparentibus deduci poſſit; ‘ ſeu ut oſtendatur ratio efficiendi per refractiones, ut tota aliqua ‘ ſuperficies certum colorem oſtendat.

‘ In librorum apud Anglos editorum indicibus occutrêre mihi ‘ aliquoties libri Mathematici autore Newtono, ſed dubitavi à Te eſſent, ‘ quod vellem, an ab alio homonymo.

‘ Heinſius noſter redux teſtis fuit benevolentiæ erga me Tuæ. De ‘ cultu verò meo erga Te, non ille tantum teſtari poteſt, ſed & ‘ Stepneius, tecum ejuſdem olim Collegii habitator, nunc magnæ ‘ Britanniæ Regis negotia apud Cæſarem, nuper apud Sereniſſimum ‘ Electorem Brandeburgicum, curans.

‘ Hæc ſcribo magis ut ſtudia erga Te mea intelligas, quæ nihil tot ‘ annorum ſilentio amiſere, quam ut Tua ego ſtudia, quibus auges ‘ humani generis opes, interrumpere velim vacuis literis & ſupervacuis. ‘ Vale. *Dabam Hanoveræ* $\frac{7}{17}$ *Martii* 1693.

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## *Part of a Letter of Dr.* Wallis*'s to Mr.* Newton, *dated from* Oxford, *April* 10*th* 1695.

‘ I Wiſh you would print the two large Letters of *June* and *October* ‘ 1676. I had Intimation from *Holland*, as deſired there by your ‘ Friends, that ſomewhat of that kind were done, becauſe your ‘ Notions of Fluxions paſs there with great Applauſe by the Name

‘ of *Leibnitz*'s *Calculus Differentialis*. I had this Intimation when all
‘ but Part of the *Preface* to this Volume was printed off; ſo that I
‘ could only inſert (while the Preſs ſtay'd) that ſhort Intimation thereof,
‘ which you there find. You are not ſo kind to your Reputation (and
‘ that of the Nation) as you might be, when you let things of worth
‘ lie by you ſo long, till others carry away the Reputation which is
‘ due to you. I have endeavoured to do you Juſtice in that, and am
‘ now ſorry that I did not print thoſe two Letters *verbatim*.

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## *The Intimation above-mentioned publiſhed in* April *1695, by Dr. Wallis in his ſaid* Preface.

QUÆ in ſecundo Volumine habentur, in Præfatione eidem præfixa dicitur. Ubi (inter alia) habetur *Newtoni* Methodus de *Fluxionibus* (ut ille loquitur,) conſimilis Naturæ cum *Leibnitii* (ut hic loquitur) *Calculo Differentiali*, (quod qui utramque methodum contulerit, ſatis animadvertat, utut ſub loquendi formulis diverſis,) quam ego deſcripſi (*Algebræ cap.* 91. &c. præſertim *cap.* 95.) ex binis *Newtoni* literis (aut earum alteris) *Junii* 13, & *Octob.* 24, 1676, ad *Oldenburgium* datis, cum *Leibnitio* tum communicandis (iiſdem fere verbis, ſaltem leviter mutatis, quæ in illis literis habentur;) ubi methodum hanc *Leibnitio* exponit, tum ante decem annos, nedum plures, ab ipſo excogitatam. Quod moneo nequis cauſetur, de hoc *Calculo Differentiali* nihil à nobis dictum eſſe.

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## *Out of the Account given of the Works of Dr.* Wallis *in the* Acta Eruditorum *for* June *1696.* Pag. 257, 258.

CÆterum ipſe *Newtonus*, non minus candore quam præclaris in rem Mathematicam meritis inſignis, publicè & privatim agnovit, *Leibnitium* tum, cum (interveniente celeberrimo Viro *Henrico Oldenburgio Bremenſi* Societatis Regiæ *Anglicanæ* tunc Secretario) inter ipſos (ejuſdem jam tum Societatis Socios) Commercium intercederet, id eſt jam fere ante annos viginti & amplius, Calculum ſuum differentialem, Serieſque infinitas, & pro iis quoque methodos generales habuiſſe; quod *Walliſius*, in Præfatione Operum, factæ inter eos communicationis mentionem faciens, præteriit, quoniam de eo fortaſſe non ſatis ipſi conſtabat. Cæterum Differentiarum conſideratio Leibnitiana, cujus mentionem facit *Walliſius*, (nequis ſcilicet, ut ipſe ait, cauſaretur de Calculo differentiali nihil ab ipſo dictum fuiſſe) meditationes aperuit, quæ aliunde non æque naſcebantur. Eſt enim Differentia Analyticum quiddam, & Calculi capax, & quod rei caput eſt, Summæ reciprocum.

*N. B.* In my Letters of the 13th of *June*, and the 24th of *October* 1676, I affirmed, That I had the Method of Fluxions ſome Years before; but I never allowed that Mr. *Leibnitz* had the Differential Method before the Year 1677, nor in thoſe Days did I know more of his Pretences to it than what he repreſented that Year in his Letter of the 21ſt of *June*; nor did I allow the Method of Tranſmutations to be a general Method of *Series*, nor then knew that the *Series* which he ſent to me, was ſent to him by Mr. *Oldenbourg* the Year before, and invented by Mr. *James Gregory* in the Year 1671. The Method of Tranſmutations is not a Method of *Series*, but a particular Theorem for Tranſmutations of Figures into one another, like thoſe of *Gregory* and *Barrow*. And as for the *Scholium* upon the ſecond *Lemma* of the ſecond Book of the *Principia Philoſophiæ Mathematica*, which is ſo much wreſted againſt me; it was written not to give away that *Lemma* to Mr. *Leibnitz*, but on the contrary to aſſert it to my ſelf. Whether Mr. *Leibnitz* invented it after me, or had it from me, is a Queſtion of no Conſequence; for ſecond Inventors have no Right.

Dr. *Wallis*, by his Letter of *Decemb.* 1. 1696, printed in the third Volume of his Works, gave Notice to Mr. *Leibnitz* of the Paragraph in his *Preface* to the firſt Volume above recited. And Mr. *Leibnitz* denied not that in the Year 1676, I explained to him the Method found by me ten Years before or above, nor complain'd of the Dr. for ſaying this. *De te autem queri* (ſaith he in his Letter of the 29th of *March* 1697,) *nunquam mihi in mentem venit, quem facile apparet noſtra in Actis Lipſienſibus prodita non ſatis vidiſſe.* He allowed that the Methods were of like Nature as the Dr. had affirmed, and ſaid (in his Letter of the 28th of *May* 1697) that he therefore called them both by the common Name of the *Infiniteſimal Method*; but added, That as the *Analyſis* of *Vieta* and *Cartes* were both called by the common Name of *Analyſis Specioſa*, and yet differ'd in ſome Things, ſo my *Infiniteſimal Analyſis* and his might differ in ſome Things, meaning ſome Improvements made by him to this Method; and excus'd the Dr. for not mentioning theſe Improvements, becauſe he had not ſufficiently ſeen them in the *Acta Lipſienſia*; and declar'd that it never came into his Mind to complain of him for any thing elſe.

And yet altho' *Cartes* never laid Claim to the *Analyſis Specioſa* of *Vieta*, Mr. *Leibnitz* continu'd amongſt his Friends to call the Infiniteſimal Method his own, and thereby gave Occaſion to Mr. *Fatio* to write what follows.

*Out*

## *Out of the Tract of* Mr. Nicholas Fatio de Duillier, *intituled*, Inveſtigatio Geometrica Solidi rotundi, in quod minima fiat reſiſtentia, *publiſhed in the Year* 1699.

‘Quæret forſan *Cl. Leibnitius*, unde mihi cognitus ſit iſte Calculus ‘quo utor. Ejus equidem fundamenta, ac pleraſque Regulas ‘proprio Marte, Anno 1687, circa menſem Aprilem & ſequentes, a‘liiſque deinceps annis, inveni; quo tempore neminem eo Calculi gene‘re præter meipſum, uti putabam. Nec mihi minus cognitus foret, ‘ſi nondum natus eſſet *Leibnitius*. Aliis igitur glorietur Diſcipulis, me ‘certe non poteſt. Quod ſatis patebit, ſi olim Literæ quæ inter cla‘riſſimum *Hugenium* meque interceſſerunt, publici juris fiant. *New‘tonum* tamen primum ac pluribus annis vetuſtiſſimum hujus Calculi ‘Inventorem, ipſa rerum evidentia coactus agnoſco: A quo utrum ‘quicquam mutuatus ſit *Leibnitius*, ſecundus ejus Inventor, malo eo‘rum quam meum ſit judicium, quibus viſæ fuerint *Newtoni* Literæ, ‘aliique ejuſdem manuſcripti Codices. Neque modeſtioris *Newtoni* ‘ſilentium, aut prona *Leibnitii* ſedulitas inventionem hujus Calculi ſibi ‘paſſim tribuentis, ullis imponet, qui ea pertractarint, quæ ipſe evolvi, ‘Inſtrumenta.

*N. B.* Mr. *Fatio* wrote this as a Witneſs. He related what he had ſeen, and his Teſtimony is the ſtronger, becauſe it was againſt himſelf, and he was no *Engliſhman*. He underſtood the Methods of us all, and by what he had ſeen and underſtood, he was able to make a true Judgment.

*FINIS.*

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## Emendanda in Epiſtolis.

PAG. 3, lin. 2, pro *habitu*, lege, *lubitu*: Pag. 112, lin ult. pro *Series*, lege, *Analyſis*. Pag. 114, lin. 18, lege, *a Battel between what he calls my forlorn Hope, and the Army of Diſciples in whom he boaſts himſelf happy*. Pag. 115, lin. 35, lege, 1673. Pag. 117, lin. 21, dele, (*thoſe of the Linea celerrimi deſcenſus, and the Linea Catenaria & Velaria*.)

In Hiſtoria Fluxionum, Pag. 92, lin. 22, pro *Menſe Octobris*, lege, *Anno*. Pag. 93. lin. 27, lege, *quam Gregorius*. Pag. 94, lin. 30, lege, *D. Collins initio*. Pag. 95, lin. 1. lege, *Perplura*. Ib. lin. 10, pro *Societatis Regiæ*, lege, *Conceſſus Arbitrorum delectorum a Societate Regia*. Ib. lin. 41, lege, 10 *Decem*. 1672. Pag. 96, lin. 11, lege, *quantitates*. Ib. lin. 12, lege, *Fluxiones*. Ib. lin. 16, lege, *habuere*. Cætera, cùm Author emortuus ſit, & Liber annis abhinc plus tribus impreſſus fuerit, emendet Lector.